Differential-Revelation VCG Mechanisms for Combinatorial Auctions

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Abstract. Combinatorial auctions, where bidders can submit bids on bundles of items, are economically efficient mechanisms for selling items to bidders, and are attractive when the bidders’ valuations on bundles exhibit complementarity and/or substitutability. Determining the winners in such auctions is a complex optimization problem that has received considerable research attention during the last 4 years. An equally important problem, which has only recently started to receive attention, is that of eliciting the bidders’ preferences so that they do not have to bid on all combinations \(^6\). Preference elicitation has been shown to be remarkably effective in reducing revelation \(^13\). In this paper we introduce a new family of preference elicitation algorithms. The algorithms in this family do not rely on absolute bids, but rather on relative (differential) value information. This holds the promise to reduce the revelation of the bidders’ valuations even further. We develop a differential-elicitation algorithm that finds the efficient allocation of items to the bidders, and as a side-effect, the Vickrey payments (which make truthful bidding incentive compatible). We also present two auction mechanisms that use differential elicitation: the difference mechanism and the difference increment mechanism.

1 Introduction

Combinatorial auctions, where bidders can submit bids on bundles of items, are economically efficient mechanisms for selling \(m\) items to bidders, and are attractive when the bidders’ valuations on bundles exhibit complementarity (a bundle of items is worth more than the sum of its parts) and/or substitutability (a bundle is worth less than the sum of its parts). Determining the winners in such auctions is a complex optimization problem that has recently received considerable attention (e.g., \([20, 24, 9, 25, 15, 12, 6]\)).

An equally important problem, which has received much less attention, is that of bidding. There are \(2^m - 1\) bundles, and each bidder may need to bid on all of them to fully express its preferences. This can be undesirable for any of several reasons: (1a) determining one’s valuation for any given bundle can be computationally intractable \([21, 23, 17, 14]\); (1b) there is a huge number of bundles to evaluate; (2) communicating the bids can incur prohibitive overhead (e.g., network traffic); and (3) bidders may prefer

\(^*\) Dr. Sandholm’s work was funded by, and conducted at, CombineNet, Inc., 311 S. Craig St., Pittsburgh, PA 15213.
not to reveal all of their valuation information due to reasons of privacy or long-term competitiveness. Appropriate bidding languages [24, 22, 5, 12] can potentially solve the communication overhead in some cases (when the bidder’s utility function is compressible). However, they still require the bidders to completely determine and transmit their valuation functions and as such do not solve all the issues. So in practice, when the number of items for sale is even moderate, the bidders will not bid on all bundles.

We study the setting in which a benevolent auctioneer (or arbitrator) wants to implement an efficient allocation of a set of heterogeneous, indivisible goods. The preferences of the participating bidders (or consumers) are private information and utility is transferable via money. The auctioneer tries to design a mechanism that gives no incentive for the bidders to misreport preferences.

It is well known that the Vickrey-Clarke-Groves mechanism [27, 5, 10] (aka. Generalized Vickrey Auction (GVA)), that is based on exhaustively eliciting all utilities, is such an incentive compatible mechanism. However, in that mechanism, each bidder evaluates each of the exponentially many bundles, and communicates a value for each one. This clearly is impractical even for auctions with moderate numbers of goods.

Consider the following: the (rational) preferences of bidders can be ranked (from most preferred towards least preferred). Each rank uniquely represents a bundle (bundles with consecutive ranks may have identical valuations). Combining the individual ranks will lead to combinations of ranks (respectively combinations of ranked bundles); some of them are feasible. All combinations form a lattice along a (weak) dominance relation. This lattice structure can be utilized to guide a (best-first) search through the space of (feasible and infeasible) combinations. This idea has been exploited in [6, 8] to design an efficient, (individually) incentive compatible mechanism for combinatorial auctions. The mechanism may reduce the amount of elicited information in comparison to generalized Vickrey auctions. The mechanism asks each bidder for the (true) valuations of (a subset of) the bundles. We called this a partial-revelation mechanism. Recently, it has been shown that this method, and related elicitation methods, may lead to significant savings in the amount of information that is elicited from the bidders (compared to the full elicitation of the GVA)—in fact, because the number of items in the auction grows, only a vanishing fraction of all value queries end up being asked [13].

In this paper we present a mechanism that does not elicit absolute valuations but rather elicits differences between valuations and, thus, may reveal only a fraction of each value information to the auctioneer. We call this differential revelation (because only differences of valuations are revealed). We present an algorithm to explore the rank lattice using differential value information. The algorithm determines an efficient allocation based on revealed valuation differentials. It also inherits the partial revelation properties of the algorithm discussed in [8], while saving the bidder from specifying absolute valuations. Note that in the worst-case, all valuation information has to be revealed—it is, however, a challenge to reduce this amount whenever possible. The algorithm was designed with this objective.

1 In general, preference communication in combinatorial auctions is provably exponential (even to find an approximately optimal solution) in the theoretical worst case [16].

2 This may be especially useful in settings where the communication between the bidder and the auctioneer is public.