Online Call Admission in Optical Networks  
with Larger Demands

Sven Oliver Krumke* and Diana Poensgen

Konrad-Zuse-Zentrum für Informationstechnik Berlin  
Takustr. 7, 14195 Berlin, Germany  
{krumke,poensgen}@zib.de

Abstract. In the problem of Online Call Admission in Optical Networks, briefly called OCA, we are given a graph $G = (V,E)$ together with a set of wavelengths $W$ and a finite sequence $\sigma = r_1, r_2, \ldots$ of calls which arrive in an online fashion. Each call $r_j$ specifies a pair of nodes to be connected and an integral demand indicating the number of required lightpaths. A lightpath is a path in $G$ together with a wavelength $\lambda \in W$. Upon arrival of a call, an online algorithm must decide immediately and irrevocably whether to accept or to reject the call without any knowledge of calls which appear later in the sequence. If the call is accepted, the algorithm must provide the requested number of lightpaths to connect the specified nodes. The essential restriction is the wavelength conflict constraint: each wavelength is available only once per edge, which implies that two lightpaths sharing an edge must have different wavelengths. Each accepted call contributes a benefit equal to its demand to the overall profit. The objective in OCA is to maximize the overall profit.

Competitive algorithms for OCA have been known for the special case where every call requests just a single lightpath. In this paper we present the first competitive online algorithms for the more general case in which the demand of a call may be as large as $|W|$.

1 Introduction

In current telecommunication networks, the wavelength division multiplexing technique (WDM) enables the provider to send several optical signals in parallel over the same glass fiber cable by assigning different wavelengths to them. However, the optical signals are converted back into electronic form at intermediate nodes in order to switch them. This so-called “o-e-o-conversion” limits the speed of the connections. In next generation’s fully optical networks, optical signals are no longer converted back into electronic form at intermediate nodes but switched optically. This requires a change in the underlying mathematical model, because the wavelength on which a signal enters the network remains unchanged until the signal reaches its destination.

A connection in a fully optical network is modeled as a lightpath, that is, a path together with a wavelength. Since each wavelength is available only once

* Research supported by the German Science Foundation (DFG, grant GR 883/10)
per fiber, simultaneously routed lightpaths which use the same fiber must have different wavelengths. This crucial restriction is called the \textit{wavelength conflict constraint}.

\subsection*{1.1 Problem Definition}

An instance of the \textit{Online Call Admission Problem in Optical Networks (OCA)} consists of an undirected graph \(G = (V, E)\) together with a set of \(\chi\) eligible wavelengths \(W = \{\lambda_1, \ldots, \lambda_\chi\}\) and a finite request sequence \(\sigma = r_1, r_2, \ldots, r_m\) of calls. Each of the wavelengths in \(W\) is available once per edge. A \textit{lightpath} is a pair \((P, \lambda)\), where \(P\) is a path in \(G\) and \(\lambda\) is one of the wavelengths in \(W\). In the sequel, we will use the terms wavelength and color interchangeably.

A \textit{call} \(r_j = (s_j, t_j, b_j)\) specifies the nodes \(s_j \in V\) and \(t_j \in V\) to be connected as well as the required number \(b_j \in \mathbb{N}\) of lightpaths, that is, its \textit{demand}. Upon arrival of a new request \(r_j = (s_j, t_j, b_j)\), an algorithm for OCA must decide whether to route or to reject \(r_j\). If the call is accepted, the algorithm must provide the requested number \(b_j\) of lightpaths, thereby obeying the wavelength conflict constraint. Once accepted, a call can not be preempted: the lightpaths used for the call can not be changed or removed anymore. Each accepted call \(r_j\) contributes a profit equal to its demand \(b_j\) to the total profit obtained by an algorithm. The overall goal of OCA is to maximize the overall profit, that is, the total accepted demand.

An \textit{online algorithm} for OCA must base its decision for call \(r_j\) without knowledge of calls \(r_i\) with \(i > j\). A standard tool to measure the quality of an online algorithm \(\text{ALG}\) is \textit{competitive analysis}, where one compares for each input sequence \(\sigma\) the profit \(\text{ALG}(\sigma)\) obtained by \(\text{ALG}\) to the optimal profit achievable on that sequence, denoted by \(\text{OPT}(\sigma)\).

\begin{definition} \textit{(Competitive Deterministic Algorithm).} \label{def:competitive_density}
A deterministic online algorithm \(\text{ALG}\) for OCA is \(c\)-competitive if for any request sequence \(\sigma\) the inequality \(\text{ALG}(\sigma) \geq \frac{1}{c} \cdot \text{OPT}(\sigma)\) holds.
\end{definition}

A randomized online algorithm is a probability distribution over a set of deterministic online algorithms. Thus, the objective value produced by a randomized algorithm is a random variable. In this paper we analyze the performance of randomized online algorithms against an \textit{oblivious adversary}. An oblivious adversary knows the online algorithm’s probability distribution, but can not see the outcomes of the random choices made by the online algorithm and therefore has to generate a request sequence in advance. We refer to [4] for details on the various adversary models.

\begin{definition} \textit{(Competitive Randomized Algorithm).} \label{def:competitive_randomized}
A randomized online algorithm \(\text{RALG}\) for OCA is defined to be \(c\)-competitive against an oblivious adversary if for any request sequence \(\sigma\) the inequality \(E[\text{RALG}(\sigma)] \geq \frac{1}{c} \cdot \text{OPT}(\sigma)\) holds.
\end{definition}

The \textit{competitive ratio} of an algorithm is defined to be the infimum over all \(c\) such that the algorithm is \(c\)-competitive.