7 Magnetoelastic (In)stability and Vibrations

7.1 Introduction

In modern technological equipments, such as fusion reactors, magnetic energy storage devices, MRI-scanners, superconducting generators, and magnetically levitated trains, huge magnetic fields occur. These fields are generated by high currents through superconducting coils. In these devices, the superconducting currents are so high that the coils are subjected to strong magnetic forces. These forces can cause unwanted vibrations (resulting in loud unpleasant noise as in MRI-scanners) or even collapse (buckling) of structures (coils) in e.g. fusion reactors. Therefore, in the design of these devices the analysis of the vibrations and the stability of magnetic and superconducting structural elements due to electromagnetic forces plays an important role.

In this chapter, we consider the (in)stability of (systems of) ferromagnetic bodies placed in an external magnetic field and of superconducting structures loaded by Lorentz forces due to the electric currents in these conductors. Since a stability problem is always an essentially nonlinear problem, the theory for it must be built upon a nonlinear set of equations for a magnetoelastic model. As seen in the first part of this book, several such models exist and thus one specific model must be chosen first. After that the general approach to the problem could run as follows: the final deformed state is considered as a perturbation of an intermediate state, for which in general the rigid-body state may be taken, and the fields in the deformed state are linearized with respect to the intermediate state. When the resulting homogeneous linear system has a non-trivial solution, we say that the intermediate state is unstable.

We start this chapter with a historical review of magnetoelastic buckling problems, in which we recapitulate some earlier results from the literature. Section 7.3 deals with ferromagnetic systems. We introduce both the so-called classical method and a variational method. Both methods are illustrated by examples dealing with a cantilevered beam of (narrow) rectangular or elliptic cross-section and a set of two parallel rods. In Sect. 7.4 the buckling of superconducting structures is treated. A variational method is introduced and illustrated by examples such as sets of two parallel rods or two concentric or parallel rings. The results are compared with the results of the so-called direct Biot–Savard method. It turns out that the latter method delivers a lower bound for the critical buckling current, which however can deviate...
substantially from the value obtained by the more exact variational method, when the slender rod-like superconductors are too close to each other. More results for superconducting structures such as helical and spiral coils are presented in Sect. 7.5. In the latter section a somewhat modified method is proposed in which the law of Biot and Savard is used to construct an admissible magnetic field for the variational method; we call this approach the combined method. Finally, Sect. 7.6 deals with magnetoelastic vibrations of magnetoelastic or superconducting systems. Eigenfrequencies are determined both by a direct method (comparable with the classical method for buckling) and by a variational method (a generalisation of Rayleigh’s principle to include magnetoelastic interactions).

7.2 Historical Review of Magnetoelastic Buckling Problems

Magnetoelastic buckling is a phenomenon in which an elastic structure becomes unstable (buckles) under electromagnetic loading. Such a structure can be, for instance, ferromagnetic or (super)conducting. The first investigations of technical relevance in this field are those of Moon [156]. He considered both ferromagnetic and conducting systems, the latter in cooperation with Chattopadhyay; see [152, 40]. A more fundamental theory of magnetoelastic stability was presented by Alblas in [9]. In this respect, also the works of Eringen [69] and of Goudjo and Maugin [76], who investigated the instability of ferromagnetic plates, should be mentioned. This subject was also studied by Van de Ven in [250].

Of more recent date is the paper of Zhou, Zheng and Miya [282], who looked at magnetoelastic buckling of ferromagnetic plates with regard to the safety of the first walls, or blankets, of a fusion reactor. These plates are of ferritic stainless steel, and thus ferromagnetic, and the interactions with the magnetic fields in the reactor are so strong that magnetoelastic buckling comes into sight. In 1997, Zhou and Zheng [283] examined the magnetoelastic instability and the increase of natural frequency of a ferromagnetic plate in a magnetic field. They developed a variational formalism by use of which they derived one general expression for the magnetic force that covers both the stability and the frequency problem at one time. Yang [273] considered a special subject, namely the buckling of a piezoelectric plate; he found a buckling load that was greater than the purely mechanical buckling load, implying that neglect of piezoelectric coupling would yield a conservative estimate of the buckling load.

Moon and Pao [156] studied the buckling problem of a cantilevered ferromagnetic beam of narrow rectangular cross-section placed in a transverse magnetic field $B_0$. They found the magnitude of the buckling field $B_{0\text{cr}}$ to be proportional to the $(3/2)$-power of the thickness-to-length ratio. This result, however, was in disagreement with their own experimental results. This