On the Inapproximability of Broadcasting Time
(Extended Abstract)

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Abstract. We investigate the problem of broadcasting information in a
given undirected network. At the beginning information is given at some
processors, called sources. Within each time unit step every informed
processor can inform only one neighboring processor. The broadcasting
problem is to determine the length of the shortest broadcasting schedule
for a network, called the broadcasting time of the network.

We show that there is no efficient approximation algorithm for the broad-
casting time of a network with a single source unless \( P = NP \). More
formally, it is \( NP \)-hard to distinguish between graphs \( G = (V,E) \) with
broadcasting time smaller than \( b \in \Theta(\sqrt{|V|}) \) and larger than \((\frac{2}{37} - \epsilon)b\)
for any \( \epsilon > 0 \).

For ternary graphs it is \( NP \)-hard to decide whether the broadcasting
time is \( b \in \Theta(\log |V|) \) or \( b + \Theta(\sqrt{b}) \) in the case of multiple sources.

For ternary networks with single sources, it is \( NP \)-hard to distinguish
between graphs with broadcasting time smaller than \( b \in \Theta(\sqrt{|V|}) \) and
larger than \( b + c\sqrt{\log b} \).

We prove these statements by polynomial time reductions from E3-SAT.

Classification: Computational complexity, inapproximability, network
communication.

1 Introduction

Broadcasting reflects the sequential and parallel aspects of disseminating in-
formation in a network. At the beginning the information is available only at some
sources. The goal is to inform all nodes of the given network. Every node may
inform another neighboring node after a certain switching time. Along the edges
there may be a delay, too. Throughout this abstract the switching time is one
time unit and edges do not delay information. This model is called Telephone
model and represents the broadcasting model in its original setting [GaJo79].

The restriction of the broadcasting problem to only one information source \( v_0 \)
has often been considered, here called single source broadcasting problem (SB).

Note that the broadcasting time \( b(G,v_0) \) is at least \( \log_2 |V| \) for a graph \( G = (V,E) \), since during each round the number of informed vertices can at most

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The smallest graph providing this lower bound is a binomial tree $F_n$ consisting of a single node and $F_{n+1}$ consists of disjunct subtrees $F_0, \ldots, F_n$ whose roots $r_0, \ldots, r_n$ are connected to the new root $r_{n+1}$. Also the hyper-cube $C_n = \{0,1\}^n$, $\{w_0v, w_1v\} \mid w, v \in \{0,1\}^*$ has this minimum broadcasting time since binomial trees can be derived by deleting edges.

The upper bound on $b(G)$ is $|V| - 1$, which is needed for the chain graph representing maximum sequential delay (Fig. 1) and the star graph (Fig. 2) producing maximum parallel delay. The topology of the processor network highly influences the broadcasting time and much effort was given to the question how to design networks optimized for broadcasting, see [LP88, BHLP92, HHL88].

Throughout this paper the communication network and the information sources are given and the task is to find an efficient broadcasting schedule. The original problem deals with single sources and its decision problem, called SBD, to decide whether the broadcasting time is less or equal a given deadline $T_0$, is $\mathcal{NP}$-complete [GaJo79, SCH81]. Slater et al. also show, for the special case of trees, that a divide-and-conquer strategy leads to a linear time algorithm. This result can be generalized for graphs with a small tree-width according to a tree decomposition of the edges [JRS98]. However, SBD remains $\mathcal{NP}$-complete even for the restricted case of ternary planar graphs or ternary graphs with logarithmic depth [JRS98].

Bar-Noy et al. [BGNS98] present a polynomial-time approximation algorithm for the single source broadcasting problem (SB) with an approximation factor of $O(\log |V|)$ for a graph $G = (V, E)$. SB is approximable within $O(\log |V|)$ if the graph has bounded tree-width with respect to the standard tree decomposition [MRSR95].

Adding more information sources leads to the multiple source broadcasting problem (MB). It is known to be $\mathcal{NP}$-complete even for constant broadcasting time, like 3 [JRS95] or 2 [Midd93]. This paper solves the open problem whether there are graphs that have a non-constant gap between the broadcasting time $b(G)$ and a polynomial time computable upper bound. In [BGNS98] this question was solved for the more general multicast model proving an inapproximability factor bound of $3 - \epsilon$ for any $\epsilon > 0$. In this model switching time and edge delay may differ for each node and instead of the whole network a specified sub-network has to be informed.

It was an open problem whether this lower bound could be transferred to the Telephone model. In this paper, we solve this problem using a polynomial time reduction from E3-SAT to SB. The essential idea makes use of the high degree of the reduction graph’s source. A good broadcasting strategy has to make most of its choices there and we show that this is equivalent to assigning variables of an E3-CNF-formula. A careful book-keeping of the broadcasting times of certain nodes representing literals and clauses gives the lower bound of $\frac{27}{2\epsilon} - \epsilon$.

We show for ternary graphs and multiple sources that graphs with a broadcasting time $b \in \Theta(\log |V|)$ cannot be distinguished from those with broadcasting time $b + c\sqrt{b}$ for some constant $c$. This result implies that it is $\mathcal{NP}$-hard to