Region Based Synthesis of P/T-Nets and Its Potential Applications

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Abstract. This talk is an informal presentation of ideas put forward by Badouel, Bernardiello, Caillaud and me for solving various types of P/T-net synthesis problems, with hints at the potential role of net synthesis in distributed software and distributed control. The ideas are theirs as much as mine. The lead is to start from Ehrenfeucht and Rozenberg’s axiomatic characterization of behaviours of elementary nets, based on regions, to adapt the characterization to P/T-nets in line with Mukund’s extended regions with integer values, and to profit from algebraic properties of graphs and languages for converting decision problems about regions to linear algebra.

1 Nets and Regions

Petri nets may be presented as matrices: rows are places, columns are events, and entries define relations between places and events. A simple net \( N \) with set of places \( P \) and set of events \( E \) is a matrix \( N : P \times E \rightarrow E \) where \( E \) enumerates all possible relations between places and events in some fixed class of Petri nets.

Thus for the P/T-nets, \( E = \mathbb{N} \times \mathbb{N} \) and \( N(p, e) = (w_1, w_2) \) represents two arcs: one arc weighted \( w_1 \) from \( p \) to \( e \), and one arc weighted \( w_2 \) from \( e \) to \( p \).

On this basis, the dynamics of Petri nets may be described in a uniform way. Given a class of nets, let \( \mathcal{N} \) be the net in this class with a single place \( \pi \) and with set of events \( E \) such that \( N(\pi, e) = e \) for every \( e \in E \) (thus \( \mathcal{N} : \{\pi\} \times E \rightarrow E \)). The state graph of \( \mathcal{N} \), let \( \mathcal{T} = (S, E, T) \) with \( T \subseteq S \times E \times S \), determines the state graph of any other net in the class. First, it defines the set \( S \) of all possible values for places. Second, an event \( e \in E \) of a net \( N : P \times E \rightarrow \mathcal{E} \) has concession at marking \( M : P \rightarrow S \) if and only if, for each \( p \in P \), the event \( N(p, e) \in \mathcal{E} \) has concession at the marking of the net \( N \) such that place \( \pi \) holds \( M(p) \); and then \( M \xrightarrow{e} M' \) where \( M' \) is the marking of \( N \) such that, for each \( p \in P \), \( M(p) \xrightarrow{N(p,e)} M'(p) \) is a transition in \( \mathcal{T} \).

Thus for the P/T-nets, \( \mathcal{T} = (S, E, T) \) is the infinite graph defined with \( S = \mathbb{N} \), \( E = \mathbb{N} \times \mathbb{N} \), and \( T = \{ m \xrightarrow{(w_1, w_2)} m' \mid m \geq w_1 \land m' = m - w_1 + w_2 \} \).
The state graph of a net \( N : P \times E \to \mathcal{E} \) embeds into the synchronous product \( T^P \) of \( |P| \) copies of \( T \): given \( e \in E \) and markings \( M \) and \( M' \), \( M \xrightarrow{e} M' \) if and only if this transition projects for each place \( p \in P \) to a transition \( M(p) \xrightarrow{N(p,e)} M'(p) \) in \( T \); and since \( e \) is the (column) vector with entries \( N(p,e) \), the transition \( M \xrightarrow{e} M' \) is precisely the synchronous product of its projections. Conversely, if for some vector \( \varepsilon : P \to \mathcal{E} \), \( M(p) \xrightarrow{\varepsilon(p)} M'(p) \) for all \( p \in P \), then \( M \xrightarrow{\varepsilon} M' \) is a transition of \( N \) if and only if \( \varepsilon \) is a column of the matrix \( N \), i.e. if \( \varepsilon \in E \). To sum up, the state graph of \( N \) is the Arnold-Nivat product of \( |P| \) copies of \( T \), using the columns of \( N \) as the synchronization vectors.

Let \( T = (S,E,T) \) be the state graph of \( N \). By construction of state graphs, each place \( p \) of \( N \) induces a map from \( T \) to \( T \) projecting transitions \( M \xrightarrow{e} M' \) to transitions \( M(p) \xrightarrow{\varepsilon(p)} M'(p) \). This map from \( T \) to \( T \) is determined by two maps operating respectively on states and on events, namely \( \sigma_p : S \to S \) with \( \sigma_p(M) = M(p) \) and \( \eta_p : E \to \mathcal{E} \) with \( \eta_p(e) = e(p) = N(p,e) \), hence it is a morphism of transition systems.

The net synthesis problem for transition systems consists in approximating at best a reachable transition system \( T \) (taken as input) by the reachable state graph of some initialized net in a specified class of Petri nets. Equivalently, the problem consists in approximating at best a transition system by the reachable restriction of some Arnold-Nivat product of copies of the state graph of the representative net \( N \). Approximations are from above, with \( T \leq T' \) if there exists a morphism of transition systems from \( T \) to \( T' \) that acts bijectively on events and that preserves the initial state. The best approximation (the least one) may be seen as a closure. A crucial problem is to decide when the approximation is exact, that is when \( T \) coincides (up to isomorphism) with the reachable state graph of some net.

Morphisms of transition systems from \( T = (S,E,T) \to T = (S',E,T) \) are privileged tools for solving the above described problems. A morphism from \( T \) to \( T \) is a pair \( (\sigma,\eta) \) of maps \( \sigma : S \to S \) and \( \eta : E \to \mathcal{E} \) such that \( \sigma(s) \xrightarrow{\eta(e)} \sigma(s') \) whenever \( s \xrightarrow{e} s' \). Each morphism \( (\sigma,\eta) \) determines an initialized net \( N \) whose state graph approximates \( T \): this net has a unique place \( p \), defined with \( N(p,e) = \eta(e) \) for all \( e \in E \), and its initial marking \( M_0(p) = \sigma(s_0) \) is the image of the initial state of \( T \). As a matter of fact, the state graph of a net \( N \) with a single place \( p \) approximates \( T \) if and only if this place derives from a morphism \( (\sigma,\eta) : T \to T \) as indicated. For the elementary nets, these morphisms are in bijective correspondence with Ehrenfeucht and Rozenberg’s regions. For the P/T-nets, these morphisms are essentially Mukund’s regions (although we do not consider here step transition systems): a morphism \( (\sigma,\eta) \) sends each state \( s \in S \) to a non negative integer \( \sigma(s) \) and each event \( e \in E \) to a pair of weights \((w_1, w_2)\) such that \( \sigma(s) \geq w_1 \) and \( \sigma(s') = \sigma(s) - w_1 + w_2 \) for every transition \( s \xrightarrow{e} s' \) in \( T \).