On the Separation between $k$-Party and $(k-1)$-Party Nondeterministic Message Complexities*

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Abstract. We introduce (reasonable) generalizations of the one-way uniform two-party protocols introduced in [6,7], which provide a closer relationship between communication complexity and finite automata as regular language recognizers. A superpolynomial separation between $k$-party and $(k-1)$-party message complexities of the nondeterministic model is established by exhibiting a sequence of concrete languages to witness it, thus a strong hierarchy result. As a consequence, the new model provides an essentially better lower bound method for estimating $\text{ns}(L)$, for some regular languages $L$. We remark that in the deterministic case hierarchy is not realized.

1 Introduction

Communication protocols accepting languages have been considered in [6,12,14]. The first, however, rather simple application of communication complexity in this respect was given in [5]. That for every regular language $L$, there is a positive integer $m$ s.t. a standard 2-party protocol can recognized it in at most $m$ communication.

Half a decade ago, one-way uniformity was introduced in [6,7] to communication complexity of 2-party protocols, which was started in [16] (and also in [1]). This effort was motivated by giving a closer relationship between communication complexity and finite automata.

Uniformity leads to an equality relation between the minimal size of a deterministic finite automata for $L$, $s(L)$ and its so-called deterministic message complexity, $\text{dmc}_2(L)$, while the nondeterministic message complexity of $L$, $\text{nmc}_2(L)$ remains to provide a lower bound for the minimal size of a nondeterministic finite automaton for $L$, $\text{ns}(L)$. Until today most lower bound techniques for $\text{ns}(L)$ are based on communication technique.

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In several occasions, it has been shown that there are languages whose nmc$_{2}(L)$ is much smaller than ns$(L)$ [9,10,11], though. In particular, a superpolynomial gap was demonstrated between them in [9,10].

In [3], a reasonable extension of the uniform model in [7] has been introduced. It has been observed, however, that, for some regular languages $L$, an additional processor helps in providing a better estimate of the lower bound for ns$(L)$.

This is corollary to the fact that the language PART$_n$ from [9] witnesses a similar gap between nmc$_2$(PART$_n$) and nmc$_3$(PART$_n$) [3]. The increase in complexity is due mainly to the fact that the input can be awkwardly partitioned between the participating computers.

Intuitively, for $k \geq 3$, our $k$-party model $P_k$ computes on any $w \in \Sigma^*$ always with respect to (w.r.t.) a $k$-partition of $w$ by passing messages from the left to right, beginning from the first computer $C_1$ consecutively until the last computer $C_k$. The message/decision to be transmitted by every $C_i$, $1 \leq i \leq k-1$, depends on $\alpha_i$ and the message obtained from $C_{i-1}$. Only $C_k$ will have to decide on the acceptance of the input, once a message from $C_{k-1}$ is received.

We represent a computation of our model as:

$$C : \Sigma^* \to 2^{\{0,1\}^*} \cup \{\text{accept}, \text{reject}\} \text{ s.t. } \forall w \in \Sigma^*,$$

$$m_1$$ $m_2$ $\cdots$ $m_{k-1}$ $m_f \in C(w).$$

A set of words or language is accepted by our model iff at least an accepting computation for each word $w$ in it ends with $m_f = \text{accept}$. A computation for a word which terminates with $m_f = \text{reject}$, connotes nonacceptance of such a word by our model. For $1 \leq i \leq k-1$, $m_i \in 2^{\{0,1\}^*}$.

We denoted by $M(P_k)$ the set of messages spent by $P_k$ in recognizing an $L$ and min $|M(P_k)|$ will be our message complexity for $L$.

An almost similar $k$-party model was presented by Tiwari [15], wherein for $w = xy$, the mode of communication is two-way aside from the first computer, (with $x$) who serves messages one-way to the right and the last one (with $y$) to its left. The middle ones have nothing at the start. However, our model is strictly one-way and every computer can have part of the input at the outset.

This paper aims to establish a superpolynomial gap between message complexities for all $k \geq 2$. Specifically, we will show that for all $k \geq 2$, there is a sequence of regular languages, $L(k,n)$ such that all $k$-party protocols will require a large amount of messages to recognized it, however, it has an efficient $(k-1)$-party protocol. This possibility was mentioned in [2,3].

The remainder of the paper will be as follows: We will first introduce our model and complexity measure, then we show that a language is regular iff it is accepted by some $k$-party protocol with finite message complexity (Section 2). Afterwards, we give a prelude (Section 3) on the possibility of a strong hierarchy on the nondeterministic message complexity. Finally, we prove the main result (Section 4), by showing

$$\text{nmc}_k(L(k,n)) = 2^{\Omega(\sqrt{\text{nmc}_{(k-1)}(L(k,n))})}.$$

A remark on the deterministic case is provided at the end (Section 5).