On the Axiomatizability of Ready Traces, Ready Simulation, and Failure Traces

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Abstract. We provide an answer to an open question, posed by van Glabbeek \cite{1}, regarding the axiomatizability of ready trace semantics. We prove that if the alphabet of actions is finite, then there exists a (sound and complete) finite equational axiomatization for the process algebra BCCSP modulo ready trace semantics. We prove that if the alphabet is infinite, then such an axiomatization does not exist. Furthermore, we present finite equational axiomatizations for BCCSP modulo ready simulation and failure trace semantics, for arbitrary sets of actions.

1 Introduction

Labeled transition systems constitute a fundamental model of concurrent computation, which is widely used in light of its flexibility and applicability. They model processes by explicitly describing their states and their transitions from state to state, together with the actions that produced them. Several notions of behavioral equivalence have been proposed, with the aim to identify those states of labeled transition systems that afford the same observations. The lack of consensus on what constitutes an appropriate notion of observable behavior for reactive systems has led to a large number of proposals for behavioral equivalences for concurrent processes.

Van Glabbeek \cite{4} presented the linear time - branching time spectrum of 15 behavioral equivalences for finitely branching, concrete, sequential processes. For 12 equivalences in this spectrum, van Glabbeek gave an axiomatization that is sound and complete for the process algebra BCCSP modulo such an equivalence. BCCSP is built from the nil \(0\), alternative composition \(+\), and prefixing \(a\), where \(a\) ranges over a nonempty set \(\text{Act}\) of actions.

For three equivalences, based on ready simulation \cite{37}, failure traces \cite{10} and ready traces \cite{2111}, the axiomatization in \cite{4} includes a conditional equation. For example, for failure trace and ready trace equivalence, the axiomatizations include the conditional equation

\[
I(x) = I(y) \Rightarrow a(x + y) \approx ax + ay
\]  

(1)
where $I(p)$ is the set of possible initial actions of process $p$. In [4, p. 78] it is remarked that for finite alphabets, ready simulation and failure trace equivalence do allow a finite equational axiomatization.

“As observed by Stefan Blom, if $Act$ is finite, ready simulation equivalence can be finitely axiomatized without using conditional equations or auxiliary operators. [...] If $Act$ is finite also failure trace equivalence has a finite equational axiomatization. However, it is unknown whether the same holds for ready trace equivalence.”

We present formal proofs of the observations regarding ready simulation and failure trace equivalence, for arbitrary sets of actions. The main part of this paper is devoted to answering the open question regarding ready trace equivalence.

Groote [5] introduced an infinite family of (unconditional) equations that, in the case of finitely branching processes, captures the conditional equation (1):

$$a \left( \sum_{i=1}^{n} (b_i x_i + b_i y_i) + z \right) \approx a \left( \sum_{i=1}^{n} b_i x_i + z \right) + a \left( \sum_{i=1}^{n} b_i y_i + z \right)$$

for $n \in \mathbb{Z}_{>0}$. We prove that if $Act$ consists of $k$ elements, then actually only equation (2) for the case $n = k$ is needed, together with the equations for ready trace equivalence from [4] (excluding (1)), to obtain a (sound and complete) finite equational axiomatization for BCCSP modulo ready trace equivalence. This provides an affirmative answer to van Glabbeek’s question in the case of a finite alphabet.

Van Glabbeek considers occurrences of actions in axioms as concrete action names, so that in the case of an infinite alphabet $Act$, an axiom such as (1) actually represents an infinite number of conditional equations, one for each $a \in Act$. In this paper we take such an occurrence of $a$ in an axiom to represent a variable of type $Act$, so that (1) represents a single conditional equation. With the latter interpretation of occurrences of actions in axioms, the equational axiomatizations for 11 of the equivalences in the linear time - branching time spectrum remain finite in the case of an infinite alphabet. However, the finite equational axiomatization for ready trace equivalence given in this paper works only in the case of a finite alphabet, due to the fact that for an infinite alphabet it no longer suffices to select only one equation from the family of equations (2). We prove that in the case of an infinite alphabet, BCCSP modulo ready trace equivalence does not allow a finite equational axiomatization.

Related work: For BCCSP modulo 2-nested simulation [6], which is part of the linear time-branching time spectrum, there does not exist a finite equational axiomatization [11]: not even in the case of a finite alphabet.

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