A Fuzzy Approach for Approximating Formal Concepts

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Abstract. In this paper we present a new approach for approximating concepts in the framework of formal concept analysis. We investigate two different problems. The first, given a set of features B (or a set of objects A), we are interested in finding a formal concept that approximates B (or A). The second, given a pair (A, B), where A is a set of objects and B is a set of features, we are interested in finding a formal concept that approximates (A, B). We develop algorithms for implementing the approximation techniques presented. The techniques developed in this paper use ideas from fuzzy sets. The approach we present is different and simpler than existing approaches which use rough sets.

1 Introduction

Formal concept analysis (FCA) is a mathematical framework developed by Wille and his colleagues at Darmstadt/Germany that is useful for representation and analysis of data [11]. A pair consisting of a set of objects and a set of features common to these objects is called a concept. Using the framework of FCA, concepts are structured in the form of a lattice called the concept lattice. The concept lattice is a useful tool for knowledge representation and knowledge discovery [4]. Formal concept analysis has also been applied in the area of conceptual modeling that deals with the acquisition, representation and organization of knowledge [6]. Several concept learning methods have been implemented in [1, 4, 5] using ideas from formal concept analysis.

Not every pair of a set of objects and a set of features defines a concept [11]. Furthermore, we might be faced with a situation where we have a set of features (or a set of objects) and need to find the best concept that approximates these features (or objects). For example, when a physician diagnosis a patient, he finds a disease whose symptoms are the closest to the symptoms that the patient has. In this case we can think of the symptoms as features and the diseases as objects. Another example is in the area of information retrieval where user's

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query can be understood as a set of features and the answer to the query can be understood as the set of objects that possess these features. It is therefore of fundamental importance to be able to find concept approximations regardless how little information is available.

In this paper we present a general approach for approximating concepts that uses ideas from fuzzy set theory. We first show how a set of features (or objects) can be approximated by a concept. We then extend our approach for approximating a pair of a set of objects and a set of features. Based on our approach, we present efficient algorithms for concept approximation.

The notion of concept approximation was first introduced in [7, 8] and further investigated in [9, 10]. All these approaches use rough sets as the underlying approximation model. In this paper, we use fuzzy sets as the approximation model. This approach is simpler and the approximation is presented in terms of a single formal concept as compared to two in terms of lower and upper approximations [7–10]. Moreover, the concept approximation algorithms that result from using a fuzzy set approach are simpler.

The organization of this paper is as follows. In Section 2 we give an overview of FCA results that we need for this paper. In Section 3, we show how to approximate a set of features or a set of objects. In Section 4, we show how to approximate a pair of a set of objects and a set of features. A numerical example explaining the approximation ideas is given in Section 5. Finally, a conclusion is drawn in Section 6.

2 Background

Relationships between objects and features in FCA is given in a *context* which is defined as a triple (G, M, I), where G and M are sets of objects and features (also called attributes), respectively, and $I \subseteq G \times M$. An example of a context is given in Table 1 where an "X" is placed in the ith row and jth column to indicate that the object at row i possesses the feature at column j. If object g possesses feature m, then $(g, m) \in I$ which is also written as gIm. The set of features common to a set of objects A is denoted by $\beta(A)$ and defined as $\{m \in M \mid gIm \ \forall g \in A\}$. Similarly, the set of objects possessing all the features in a set $B \subseteq M$ is denoted by $\alpha(B)$ and given by $\{g \in G \mid gIm \ \forall m \in B\}$. A *formal concept* (or simply a *concept*) in the context (G, M, I) is defined as a pair (A, B) where $A \subseteq G$, $B \subseteq M$, $\beta(A) = B$ and $\alpha(B) = A$. A is called the *extent* of the concept and B is called its *intent*. For example, the pair (A, B) where $A = \{4, 5, 8, 9, 10\}$ and $B = \{c, d, f\}$ is a formal concept. On the other hand, the pair (A, B) where $A = \{2, 3, 4\}$ and $B = \{f, h\}$ is not formal concept because $\alpha(B) \neq A$. A pair (A, B) where $A \subseteq G$ and $B \subseteq M$ which is not a formal concept is called a *non-definable concept* [10]. The Fundamental Theorem of FCA states that the set of all formal concepts on a given context with the ordering $(A_1, B_1) \leq (A_2, B_2)$ iff $A_1 \subseteq A_2$ is a complete lattice called the *concept lattice* of the context [11]. The concept lattice of the context given in Table 1 is shown in Figure 1 where concepts are labeled using *reduced labeling* [2]. The extent of a