Combining Logics: Parchments Revisited

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Abstract. In the general context of the theory of institutions, several notions of parchment and parchment morphism have been proposed as the adequate setting for combining logics. However, so far, they seem to lack one of the main advantages of the combination mechanism known as fibring: general results of transference of important logical properties from the logics being combined to the resulting fibred logic. Herein, in order to bring fibring to the institutional setting, we propose to work with the novel notion of \( c \)-parchment. We show how both free and constrained fibring can be characterized as colimits of \( c \)-parchments, and illustrate both the construction and its preservation capabilities by exploring the idea of obtaining partial equational logic by fibring equational logic with a suitable logic of partiality. Last but not least, in the restricted context of propositional based, we state and prove a collection of meaningful soundness and completeness preservation results for fibring, with respect to Hilbert-like proof-calculi.

1 Introduction

Recently, the problem of combining logics has been deserving much attention. The practical impact of a theory of logic combination is clear for anyone working in knowledge representation or in formal specification and verification. In the fields of artificial intelligence and software engineering, the need for working with several formalisms at the same time is widely recognized. Besides, combinations of logics are also of great theoretical interest [4]. Among the different combination techniques, both fibring [12,13,22] and combinations of parchments [20,21] deserve close attention. In fact, although the work on parchments has found its way into practice, see for instance [19], it lacks a feature that we consider essential: transference results for relevant properties of logics, such as soundness and completeness. For fibring, however, recent significant preservation results have been obtained [29]. Our goal in this paper is to bring both fibring and these transference results to the setting of institutions.

This leads us, first of all, to a revised notion of parchment. It shall be made clear that the detail provided by early definitions [13,20,21] is not enough to capture the finer structure of models. In particular, for a smooth characterization of fibring, we need a notion that promotes logical consequence as a whole, rather than just validity. In previous work [22,29], a validity based consequence
has also been considered and related to this more “internal” notion [7]. Herein, however, we shall not make explicit use of it. Still, the distinction is crucial to the full understanding of many logics, including first-order logic and modal logic, and plays an essential role in the process. So, we propose to work with \( c \)-parchments, that essentially extend the model-theoretic parchments of [21] by endowing the algebras of truth-values with more than just a set of designated values. Namely, we require the set of truth-values to be structured according to a Tarskian closure operation as in [7], thus recovering an early proposal of Smiley [25].

Besides showing how \( c \)-parchments can be seen as presentations of institutions, a suitable notion of morphism is also proposed and shown to present institution op-morphisms. The reason for this relationship to the dual of the category of institutions and institution morphisms is precisely our intention to follow the “old slogan” in its strict sense, and use colimits for combination. Therefore, building on the fact that \( c \)-parchments are essentially functors over a suitable category of \( c \)-rooms, we manage to characterize both free and constrained fibring as colimits of \( c \)-parchments. We illustrate fibring by providing a detailed construction of an equational logic dealing with partiality, by combining equational logic with a suitable logic of partiality. This example, when compared with the way partiality is dealt with using previous notions of parchment [20,21,19], is in fact paradigmatic of the modular power of fibring. Along with the fibred semantics of partial equational logic, we also show that by simultaneously combining Hilbert-like proof-calculi for the given logics, a sound and complete calculus for partial equational logic can also be obtained.

In fact, given that the right amount of structurality [6] is embodied in the deduction rules of proof-calculi, their fibring is well understood [7] and meaningful. As in previous treatments of this issue, we shall achieve this by using schema variables to write schema rules that can then be instantiated with arbitrary formulae while building deductions. In this context, although just for the particular case of propositional based logics, we then state and prove a collection of soundness and completeness transference results for fibring. Preservation of soundness is easily just a consequence of the construction underlying fibring, as shown in [7]. On the contrary, as should be expected, completeness preservation results are in general not so easy to obtain. The completeness transference results that we shall present are based on the fundamental notion of fullness, as a means of guaranteeing that we always have enough models, extending original ideas from [29], further worked out in [7]. We provide completeness proofs for several classes of interpretation structures, including partially-ordered ones, using standard techniques in logic and algebra, such as congruence and Lindenbaum-Tarski algebras. Rephrasing the main Theorem of [29], we also mention the case of powerset structures inspired by general models for modal logic (see for instance [16]) whose completeness proof uses a Henkin-style technique.

The rest of the paper is organized as follows. In Section 2 we introduce the novel notion of \( c \)-parchment and show how it relates to institutions. For the sake of illustration, we show how to represent two well known logics as \( c \)-parchments.