5 Bohm Trajectory Approach to Timing Electrons

C. Richard Leavens

Institute for Microstructural Sciences, National Research Council of Canada, Ottawa, Canada K1A 0R6

5.1 Introduction

Theoretical expressions\cite{1,2,3,4,5,6} for the 1D probability distribution $\Pi(T;X)$ of arrival times $T$ of quantum particles at $x = X$ are often written as the sum of two terms $\Pi_+(T;X)$ and $\Pi_-(T;X)$, corresponding to arrivals from the left and from the right respectively, with no left-right interference term. Thinking classically for the moment so that the possibility of such an interference term is not an issue, to say that a point particle arrives at $x = X$ at the time $t = T$ from the left (right) means that the particle is located at $x = X$ at $t = T$ and that it was in the region $x < X$ ($x > X$) at all times $t$ in the interval $T - \Delta t \leq t < T$ for some nonzero $\Delta t$. This concept is a simple, clear and meaningful one for a particle which moves along a continuous trajectory with a well-defined finite velocity at each instant of time. Returning to the quantum case, one can attempt to maintain this simplicity and clarity by adopting an approach involving continuous particle trajectories, either “virtual” or “real”, and using a relativistic wave equation to avoid infinite velocities. For the special case of electrons, considered here, at least two such approaches come to mind. One is based on Feynman’s path integral derivation\cite{7} of the 1+1 dimensional free-electron Dirac equation using a “checkerboard” model in which particles move along zig-zag paths in space-time always at the vacuum speed of light $c$. It has recently been shown\cite{8} that, when the finite correlation length for reversals of direction discovered by Jacobson and Schulman\cite{9} is taken into account, this model leads to an arrival-time distribution for free motion in 1+1 dimensions with no interference term. The results for $\Pi_+(T;X)$ and $\Pi_-(T;X)$ are identical to those presented without derivation or discussion thirty years ago by Wigner\cite{10}. In another approach that comes to mind, the trajectory method based on the causal version of Bohm’s ontological interpretation of relativistic quantum mechanics\cite{11,12,13,14,15,16}, the particle speed associated with a Dirac electron cannot exceed $c$ and there is no interference term in $\Pi(T;X)$. Both approaches are simple and clear but whether or not they are physically meaningful is wide open for discussion, particularly since the two arrival-time distributions are not identical. The first approach is based on the implicit assumption that the directed arrival-time distributions calculated using the virtual paths of the checkerboard model are identical to those for actual elec-
trons and the second on the explicit postulate that Bohm trajectories are real.

The Bohm trajectory approach is the primary focus of this chapter. In Sect. 5.2 the essentials of Bohm’s ontological interpretation of quantum theory are sketched and then applied to timing electrons. Some conventional approaches are discussed from the point of view of Bohmian mechanics in Sect. 5.3. Spin-dependent arrival-time distributions for nonrelativistic electrons are considered in Sect. 5.4. Section 5.5 addresses a recent claim based on a gedanken protective measurement that Bohm trajectories are not real. Concluding comments are made in Sect. 5.6.

5.2 Bohm’s Ontological Interpretation of Quantum Theory

5.2.1 Brief Introduction

Much of the work on characteristic times for quantum particles in terms of (assumed) real trajectories, as opposed to Feynman’s virtual paths, has been carried out within the framework of Bohm’s ontological interpretation of quantum mechanics [11,12,13,14,15,16]. In the causal version of the theory, tailored to the single-particle problem of interest here, it is postulated that an electron propagating in a potential $V(r,t)$ is an actual point-like particle and an accompanying wave $\psi(r,t)$ which probes the potential and guides the particle’s motion accordingly so that it has a deterministically well-defined position $r(t)$ and velocity $v(t)$ at each instant of time $t$. It is also postulated that the particle’s equation of motion is

$$v(t) \equiv dr(t)/dt = \frac{J(r,t)}{\rho(r,t)} \bigg|_{r=r(t)}.$$  (5.1)

For a Dirac electron (see Appendix)

$$\rho(r,t) \equiv \psi^\dagger(r,t)\psi(r,t) \ , \ J(r,t) \equiv c\psi^\dagger(r,t)\hat{\alpha}\psi(r,t),$$  (5.2)

where the 4-component guiding field $\psi(r,t)$ is the appropriate solution of Dirac’s relativistic wave equation and $c\hat{\alpha}$ is the Dirac velocity operator. We are using the minimal approach of Bell [13] in which the dynamical properties of an electron usually associated with spin follow solely from the assumption that the spatial motion of the point-like particle is guided by a

1 The local expectation value of a property represented by an operator $\hat{O}$ is defined to be $\text{Re}\left[\psi^\dagger(r,t)\hat{O}\psi(r,t)\right]/\psi^\dagger(r,t)\psi(r,t)$. Although the particle velocity $v(r,t)$ in (5.1) is identical to the local expectation value of the velocity operator $\hat{\alpha}$, the relevant expression for the square of the particle velocity is $\{[\psi^\dagger(r,t)c\hat{\alpha}\psi(r,t)]/\psi^\dagger(r,t)\psi(r,t)\}^2 \leq c^2$ rather than the local expectation value $\text{Re}\left[\psi^\dagger(r,t)(c\hat{\alpha})^2\psi(r,t)/\psi^\dagger(r,t)\psi(r,t)\right] = 3c^2$. (See Appendix.)