13 Uncertainty Bands Approach to LFT Modelling

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Summary. This chapter discusses numerical techniques for generating linear fractional transformation based uncertainty models for use in the clearance procedure of flight control systems using the structured singular value \( \mu \). The numerical techniques do not require closed-form linear expressions for the aircraft dynamics – only a non-linear software model of the closed loop aircraft, which can be efficiently trimmed and linearised numerically, is required. A refined approach for generating linear fractional transformation based uncertainty models is presented. The resulting parametric models allow to identify worst-case uncertain parameter combinations.

13.1 Introduction

In order to apply \( \mu \)-analysis techniques to the flight control law certification problem, a parametric uncertainty model which allows a particular linear fractional transformation (LFT) representation of the uncertain closed loop system must first be generated, see Fig. 13.1. \( M \) represents the known part of the system (plant and controller) and \( \Delta \) represents the uncertainty present in the system. In effect, extra inputs and outputs are introduced so that the system uncertainty can be considered as part of an “external feedback loop”. \( \mu \) defines a stability-test for a closed loop system subject to structured uncertainty \( \Delta \) in terms of the maximum structured singular value [1].

In recent years much attention has been paid to the issue of how to efficiently generate accurate (and ideally minimal) LFT-based uncertainty models for complex uncertain systems – see [2] for an overview. A common assumption among almost all of the approaches suggested is that closed form analytical expressions relating the aircraft dynamics to the uncertain parameters of interest are available, from which LFT-based uncertainty models may be derived. Such expressions usually take the form of non-linear equations of motion involving the uncertain parameters, which when linearised
symbolically using dedicated software tools [3], [4], [5], result in linear state-space models whose coefficients depend explicitly on the uncertain parameters. Given state-space models in this form, the generation of accurate, if not always minimal, LFT-based uncertainty models is then relatively straightforward – see [6], [7] and [8] for some flight control examples. The main drawbacks of the above approach can be identified as the substantial modelling effort required to accurately relate all the uncertain parameters to the non-linear aircraft dynamics, and the fact that the symbolically linearised state-space models are generally valid only at and around the relevant operating point in the flight envelope.

Here, an alternative approach for generating LFT-based uncertainty models is presented which does not require the availability of analytical expressions relating the aircraft dynamics to the uncertain parameters – only a non-linear software model of the closed loop aircraft, which can be efficiently trimmed and linearised numerically for different values of the uncertain parameters, is required. The proposed approach thus allows a significant reduction in the modelling effort required, and in general can be applied to complex systems which cannot be satisfactorily described using simple differential equation based symbolic models.

### 13.2 LFT Modelling Using Trends and Bands

#### 13.2.1 Uncertainty Modelling

As shown in [9], LFT-based parametric uncertainty models may be conveniently derived from a linear state space representation of the uncertain system of the form

\[
\begin{align*}
\dot{x} &= (A_0 + A_1 \delta_1 + \cdots + A_n \delta_n)x + (B_0 + B_1 \delta_1 + \cdots + B_n \delta_n)u \\
y &= (C_0 + C_1 \delta_1 + \cdots + C_n \delta_n)x + (D_0 + D_1 \delta_1 + \cdots + D_n \delta_n)u
\end{align*}
\]  

(13.1)

The matrices \(A_0, B_0, C_0, D_0\) describe the nominal system, while \(A_k, B_k, C_k, D_k\), with \(k = 1, \ldots, n\) describe deviations from the nominal system depending on