A New Structure of Cylinder Packing

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Abstract. We report a new periodic structure of the cylinder packing. All the cylinders are congruent and the length of the cylinders are infinite and their directions are restricted to only six directions of (110). Each cylinder is fixed by cylinders of other directions, so that the whole structure sustains mechanical stability. The packing density equals to \((\frac{\sqrt{3}}{2}-0.066/\pi) \approx 0.3762\) (\(0.494<0.247\)). The arrangement of parallel cylinders forms a certain 2D rhombic lattice common to all of six (110) directions. Nevertheless the way of fixing cylinders is different in all of six directions: the cylinders of two directions are supported with the rhombus-type, and the cylinders of other four directions are supported with the equilateral-triangle-type. The structure containing the equilateral-triangle-type has never been known.

1 Introduction

The history of the research on the cylinder packing is much shorter than that on the sphere packing. Few mathematicians have treated the problem[1]. Some simple structures of the cylinder packing appear in the books about solid puzzles(e.g. Holden[2], Coffin[3] ). O’Keeffe and Andersson applied the cylinder packing to the science[4][5][6][7]. They are crystal chemists and explained the garnet structure famous for its complexity by using a periodic cylinder packing restricted to only four directions of (111).

In engineering, some structures of the cylinder packing are used for the composite materials. Such structures are light and tough against the stress from various directions. Some periodic structures were designed (Hatta[8], Hikikata

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and Fukuta[9]). Stimulated by their researches, the authors investigated other periodic structures with six directions [10][11][12][13][14], and paid much effort exhaustively to find all the possible structures with high symmetry and density. More precisely, all the cylinders are supported by contacts of the cylinders in more than four directions among other five. Watanabe found the remarkable fact that each structure could continuously modify itself without changing its directions of cylinders, its packing density and its stability at the same time when the cylinders are kept tangent one another. An animation can be demonstrated on his web-site[14].

Quasiperiodic packings of cylinders were also reported. An architect Hizume extended Coffin's structure and Ogawa cooperated the research[15][16][13]. The structures have the icosahedral symmetry, and are composed of six directions indicated by \( \langle 170 \rangle \) \( \tau = (1 + \sqrt{5})/2 \). Other quasiperiodic packings were discussed in recent papers[17][18][19].

The structures of the cylinder packing have to be researched more systematically. The present paper suggests further possibility of \( \langle 110 \rangle \) six-axes structures. The theory of \( \langle 110 \rangle \) structure will change to general one.

2 The New \( \langle 110 \rangle \)-Structure

The structure is a six-axes periodic structure made of cylinders with one diameter \( d \). Some similar structures whose packing density are 0.494 or 0.247 have been known and are listed on Table 1 in Appendix. For the sake of convenience, we call the structures respectively Type-I(density 0.494), Type-II(density 0.247), and Type-III(the present structure).

The six directions are \( A(1,1,0), B(1,-1,0), C(1,0,1), D(-1,0,1), E(0,1,1), F(0,1,-1) \). If we classify these directions according to perpendicularly, they separate into three groups: \( A-B, C-D, \) and \( E-F \). Another classification is made by the relations of sixty degrees:

- \( B, D \) and \( F \) are sixty degrees each other. A common normal vector is \( (1,1,1) \)
- \( A, C \) and \( F \) are sixty degrees each other. Normal vector \( (-1,1,1) \)
- \( A, D \) and \( E \) are sixty degrees each other. Normal vector \( (1,-1,1) \)
- \( B, C \) and \( E \) are sixty degrees each other. Normal vector \( (1,1,-1) \)

When we care about cylinders parallel to \( A \)-direction in the structure of Type-III, they form a rhombic lattice on a plane perpendicular to \( A \)-direction. Each of other five directions also forms the rhombic lattice on a plane perpendicular to the direction. Moreover, the six rhombic lattices are congruent. A rhombus of the rhombic lattice has two diagonals, and we express that the length of the longer diagonal is \( 2a \) and that the length of the shorter diagonal is \( \sqrt{2}a \). The relation between \( a \) and \( d \) is

\[
a = (2/3)(1 + 2\sqrt{3})d.
\]  

(1)

The following expression is written with \( a \) and \( d \) to make formulae simple.