On the Spectrum and Structure of Internet Topology Graphs*

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Abstract. In this paper we study properties of the Internet topology on the autonomous system (AS) level. We find that the normalized Laplacian spectrum (*nls*) of a graph provides a concise fingerprint of the corresponding network topology. The *nls* of AS graphs remains stable over time in spite of the explosive growth of the Internet, but the *nls* of synthetic graphs obtained using the state-of-the-art topology generator Inet-2.1 is significantly different, in particular concerning the multiplicity of eigenvalue 1. We relate this multiplicity to the sizes of certain subgraphs and thus obtain a new structural classification of the nodes in the AS graphs, which is also plausible in networking terms. These findings as well as new power-law relationships discovered in the interconnection structure of the subgraphs may lead to a new generator that creates more realistic topologies by combining structural and power-law properties.

1 Introduction

Significant research efforts have recently been invested in the analysis of the Internet topology. The current Internet is the result of rapid, distributed growth without controlled planning by a central authority. Therefore, its topology reflects in great parts the choices and decisions made by individual organizations whose subnetworks form the Internet. As a consequence, the characteristics of the Internet topology can only be investigated by analyzing the available data about the current connectivity of routers or autonomous systems or snapshots of that connectivity taken at an earlier time.

Gaining additional knowledge about the properties of the Internet topology is important for several reasons. In particular, optimization problems related to resource allocation, call admission control, routing, and Distributed Denial of Service (DDoS) attack prevention (see [13]) that are provably difficult to solve for general topologies might allow efficient solutions for a class of networks containing the real Internet. Furthermore, a good understanding of the Internet topology can lead to improvements in network topology generators in order to generate "Internet-like" networks of various sizes for simulations. Network

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simulations with realistic topologies can again help to design, tune and evaluate new protocols, applications, and algorithms.

1.1 Related Work: Topology Models and Generators

Until 1999, one of the most popular network generators was GT-ITM [3], a generator that combines the hierarchical models called Transit-stub and Tiers with popular random graph models such as Waxman's model [18]. However, a major new insight into properties of the real Internet topology was gained by Faloutsos et al. [7]. They found four power-laws[1] that appear to hold for various relations between popular graph metrics in the Internet (both on the router level and on the AS level): node degree vs. node rank, degree frequency vs. degree, number of nodes within a certain number of hops vs. number of hops, and 20 largest eigenvalues of the the adjacency matrix vs. their ranks. Thus it became clear that realistic topology generators must produce graphs satisfying these power-laws.

Exploring the power-law degree distribution in WWW and Internet graphs, Barabási and Albert [1] proposed *incremental growth* – the fact that the nodes are added incrementally – and *preferential connectivity* – which means that the probability of connecting a new node to node i is proportional to the degree of i – as two main reasons for the appearance of power-laws. Based on this model, the BRITE topology generator was created [12].

Jin et al. [11] proposed a model called Inet. For a given number of nodes and percentage of nodes with degree 1, the power-law exponents from the real AS Internet graphs are used to determine the degree distribution of the resulting graph. A spanning tree using only nodes with degree at least two is created. The degree 1 nodes are then attached to the tree with proportional probability.

A generalization of the linear preference in the Barabási–Albert model and a comparison of different power-law topology generators can be found in [2].

Tangmunarunkit et al. in [15] compared structural generators based on hierarchical models (such as GT-ITM) to "purely" power-law degree-based generators (such as BRITE or Inet). Using different graph-theoretic metrics, they argued that the degree-based generator models are more realistic than structural ones and that, surprisingly, a certain hierarchical structure is present even in the degree-based generator models.

A simple model for the Internet topology consisting of five layers determined by node degrees is given in [16]. They noticed a power-law in the connection of degree-one nodes to their neighbors, which is related to our observation of other power-laws in the structure of the Internet topology (see Section 4).

A specific behavior of the spectral density of different "real-world" graphs has been noticed in [8]. They also propose spectral analysis as a promising tool for network topology classification. Correlations among nodes in the real AS Internet graphs have been studied in [14].

[1] A power-law holds between two properties y and x if y is roughly proportional to x^c for some constant exponent c. If (x, y) data pairs are plotted with both axes in logarithmic scale, the resulting points lie close to a straight line with slope c.