VSDITLU: A Verifiable Symbolic Definite Integral Table Look-Up

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Abstract. We present a verifiable symbolic definite integral table look-up: a system which matches a query, comprising a definite integral with parameters and side conditions, against an entry in a verifiable table and uses a call to a library of facts about the reals in the theorem prover PVS to aid in the transformation of the table entry into an answer. Our system is able to obtain correct answers in cases where standard techniques implemented in computer algebra systems fail. We present the full model of such a system as well as a description of our prototype implementation showing the efficacy of such a system: for example, the prototype is able to obtain correct answers in cases where computer algebra systems [CAS] do not. We extend upon Fateman's web-based table by including parametric limits of integration and queries with side conditions.

1 Introduction

In this paper we present a verifiable symbolic definite integral table look-up: a system which matches a query, comprising a definite integral with parameters and side conditions, against an entry in a verifiable table, and uses a call to a library of facts about the reals in the theorem prover PVS to aid in the transformation of the table entry into an answer. Our system is able to obtain correct answers in cases where standard techniques, such as those implemented in the computer algebra systems [CAS] Maple and Mathematica, do not. The importance of this work lies both in the novelty of verifiable table look up, and, more generally, as an indication of how theorem proving, particularly embedded verification with library support, can be a valuable tool to support users of mathematics, such as engineers, who want trusted results with minimal user interaction. NAG Ltd, the developers of the CAS **axiom**, brought this problem to our attention and are interested in including such a system in future projects.

Tables of mathematical formulae have been used by engineers and technicians for centuries. Inevitably such tables contained errors, sometimes slips of the pen, sometimes deliberate changes to foil copyists. In many cases accessible high-speed computation has allowed us to replace tables with on-the-fly calculation. For example a navigator's instruments and tables can now be replaced by an efficient GPS device, or an engineer's handbook with an "interactive book" based on a

H. Ganzinger (Ed.): CADE-16, LNAI 1632, pp. 112–126, 1999.

computer algebra system. However they are not entirely obsolete: the notorious Pentium bug was due to an error in a look-up table for SRT division, and this prompted the development by Owre and others of a general framework in the PVS prover for handling such tables [ORS97].

Definite integration, or "finding the area under a curve" is traditionally carried out using numerical techniques. However these cannot be used in the presence of parameters, and it is widely recognised in the computer algebra community [Sto91,Dav] that symbolic definite integration in the presence of parameters is a tricky problem where current algorithms are not adequate and computer algebra systems can get even very simple examples wrong. Thus table-look up is recognised as a useful solution, particularly as the answers often contain subtle side conditions. Machine look-up tables have obvious advantages over paper ones, offering automated pattern matching and simplification, ability to handle far more complex table entries and side conditions, and interoperability with other software. In particular web-based tables can be routinely updated with new results, and allow sharing and reuse of entries which may be complicated to obtain and are likely to be useful to other practitioners, as well as providing interesting opportunities for investigating user demand (which is often for nothing more difficult than homework problems). All the published paper look-up tables contain errors, so verifiable machine look up tables offer a greater possibility of freedom from error through the use of machine certification of the table entries, and their verifiable transformation to a correct answer.

In the next section we describe the problem of symbolic integration in more detail, and indicate why look-up tables are valuable. Section 3 describes the full concept for a VSDITLU, while Section 4 describes our prototype implementation, which is able to obtain correct answers in cases where standard techniques, such as those implemented in Maple and Mathematica, do not. Our prototype system extends the best available electronic table, Fateman's web-based table [EF95], by including parametric limits of integration and queries with side conditions. Section 5 addresses some of the wider issues and places our work in the context of other recent work on integrating theorem proving and computer algebra.

2 Symbolic Definite Integration

Thirty years ago an engineer wishing to compute a standard indefinite or definite integral, and preferring a trusted authority over uncertain high school math skills, would have turned to books of tables such as Gröbner and Hofreiter [GH61] or the CRC tables [ZKR96]. For example in the four hundred or so pages of [GH61] we find Entry 7 of table 1 Volume I, which expresses an indefinite integral, or more precisely an antiderivative[1], in high school math terms "a function whose

[1] Note that throughout this paper, for a a positive real, $\sqrt{a}$ denotes the positive square root of a and Log$a$ denotes the natural logarithm of a. Log denotes the principal value of the complex logarithm function.