Center Manifold Theory Approach to the Stability Analysis of Fuzzy Control Systems

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Abstract. The paper proposes a stability analysis method based on the application of the center manifold theory belonging to the state-space methods to the stability analysis of fuzzy control systems. The methods considers a linearized mathematical model of the second order nonlinear plant, and its only constraint is in the smooth character of the right-hand term of the state-space equations of the controlled plant. The method is exemplified by applying it to the stability analysis of a state feedback fuzzy control system meant for the position control of an electrohydraulic servosystem.

1 Introduction

Similar to the case of conventional control systems, a major problem which follows from the development of fuzzy controllers (briefly, FCs) is the analysis of the structural properties of the control system like stability, controllability and robustness. This results in the necessity of stability analysis methods that should ensure the global stability of the fuzzy control system (FCS) in its development phase. Furthermore, in the phase of FC implementation, the stability of the developed FCS has to be tested in different operating regimes.

For the sake of the immediately solving of the stability analysis and testing problem - and this problem captured the attention of a part of control systems specialists during the last decade according to [1] - the literature recommends several methods mainly based on the classical theory of nonlinear dynamical systems which consider that the controlled plant (CP) is modeled as a deterministic, crisp, (linear or nonlinear) system, and the FC is considered as particular case of nonlinear controller. These methods are developed in the time or frequency domain, and the most frequently encountered methods are based on:

- the state-space approach based on a linearized model of the nonlinear system [2], [3], [4], [5];
- the use of the hyperstability theory after Popov [6], [7], [8], [9], [10];
- the use of the stability theory after Lyapunov [11], [12], [4], [7], [13];
- the circle criterion [2], [4], [6];
- the harmonic balance method [14], [15], [16], [8].
The stability analysis method proposed in the paper belongs to the first category presented above, and it is based on the center manifold theory [17] and on the general theory of nonlinear control systems [18].

The paper is organized as follows. The following section deals with aspects concerning the mathematical models accepted in the stability analysis. Then, section 3 concerns with an outline of the proposed center manifold approach to the stability analysis of fuzzy control systems. Section 4 performs the application of the stability analysis method to the position control of an electrohydraulic servosystem by means of a state feedback fuzzy controller, and the final section highlights the conclusions.

2 Mathematical Models Involved in Stability Analysis

The dynamics of the controlled plant is considered described by the following state-space equation (in matrix expression):

\[ \dot{x} = f(x) + Bu, \quad (1) \]

where \( x = [x_1, x_2, ..., x_n]^T \in \mathbb{R}^n \) represents the state vector, \( b \) is an \([n, 1]\) dimensional vector of constant coefficients, \( u \) is the control signal, and \( f: \mathbb{R}^n \rightarrow \mathbb{R}^n \) stands for the process function. The only constraint imposed to \( f \) is that it must be a smooth one.

The expression (1) of the mathematical model of CP is well acknowledged and accepted for the characterization of the CP in the situations when the stability analysis of FCSs is performed [2], [5].

The relations in the sequel will be particularized for second order systems (\( n=2 \)) (the generalization to higher order systems is not a difficult task). Therefore, the relation (1) transforms into (2) and (3):

\[ x_1 = f_1(x_1, x_2) + b_1 u, \quad (2) \]
\[ x_2 = f_2(x_1, x_2) + b_2 u, \quad (3) \]

where \( f_1, f_2: \mathbb{R}^2 \rightarrow \mathbb{R} \) are smooth functions, and \( b_1, b_2 = \text{const} \in \mathbb{R} \).

Depending on the values of the constants \( b_1 \) and \( b_2 \) a coordinate transformation can be derived, and in the new coordinates the state-space mathematical model (2), (3) becomes (4), (5):

\[ x_1 = g_1(x_1, x_2), \quad (4) \]
\[ x_2 = g_2(x_1, x_2) + u, \quad (5) \]

where \( g_1, g_2: \mathbb{R}^2 \rightarrow \mathbb{R} \) are again smooth functions, and the new state variables were denoted (this is an abuse of notation, for the sake of simplicity) by \( x_1 \) and \( x_2 \).

By the application of the nonlinear state feedback control law:

\[ u = - g_2(x_1, x_2) - k_1 x_1 - k_2 x_2 - h(x_1, x_2), \quad (6) \]