The m-Cost ATSP

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Abstract. Although the m-ATSP (or multi traveling salesman problem) is well known for its importance in scheduling and vehicle routing, it has, to the best of our knowledge, never been studied polyhedraly, i.e., it has always been transformed to the standard ATSP. This transformation is valid only if the cost of an arc from node i to node j is the same for all machines. In many practical applications this is not the case, machines produce with different speeds and require different (usually sequence dependent) setup times. We present first results of a polyhedral analysis of the m-ATSP in full generality. For this we exploit the tight relation between the subproblem for one machine and the prize collecting traveling salesman problem. We show that, for $m \geq 3$ machines, all facets of the one machine subproblem also define facets of the m-ATSP polytope. In particular the inequalities corresponding to the subtour elimination constraints in the one machine subproblems are facet defining for m-ATSP for $m \geq 2$ and can be separated in polynomial time. Furthermore, they imply the subtour elimination constraints for the ATSP-problem obtained via the standard transformation for identical machines. In addition, we identify a new class of facet defining inequalities of the one machine subproblem, that are also facet defining for m-ATSP for $m \geq 2$. To illustrate the efficacy of the approach we present numerical results for a scheduling problem with non-identical machines, arising in the production of gift wrap at Herlitz PBS AG.

1 Introduction

For Herlitz PBS AG, Berlin, we are developing a software package for the following scheduling problem. Gift wrap has to be printed on two non-identical printing machines. The gift wrap is printed in up to six colors on various kinds of materials in various widths. The colors may differ considerably from one gift wrap to the next. The machines differ in speed and capabilities, not all jobs can be processed on both machines. Setup times for the machines are quite large in comparison to printing time. They depend strongly on whether material, width, or colors have to be changed, so in particular on the sequence of the jobs. The task is to find an assignment of the jobs to the machines and an ordering on these machines such that the last job is finished as soon as possible, i.e., minimize the makespan for a scheduling problem on two (non-identical) machines with sequence dependent set-up times.

G. Cornuéjols, R.E. Burkard, G.J. Woeginger (Eds.): IPCO'99, LNCS 1610, pp. 242–258, 1999.

An obvious mathematical model for this problem is the asymmetric multi traveling salesman problem with separate arc costs for each salesman. Although there is considerable literature on the TSP/ATSP [10,13,14,15,3,11,7] as well as on the m-ATSP for vehicle routing (see [5] and references therein) it seems that the m-ATSP problem in full generality has never been studied from a polyhedral point of view. Existing work on the m-ATSP relies on the standard transformation to ATSP (see e.g. [16] or Section 6) which assumes that all salesmen need the same time for the same route. The general case of non-identical salesmen should also be of importance in vehicle routing where usually not all vehicles in a car park have the same capabilities or fuel consumption.

In this paper we give a precise definition of what we call the m-Cost ATSP and present a 'canonical' integer programming formulation. This formulation includes an exponential class of inequalities, which we call the *v0-cut inequalities*. They may be interpreted as a kind of conditional subtour elimination constraints and are, in a certain sense, equivalent to the inequalities introduced in [1], Theorem 2.5, for the Price Collecting TSP (PCTSP) (see also [8,2,9]). The PCTSP is tightly related to the one machine subproblem of the m-Cost ATSP and we will clarify this relation in Section 4. Another variant, the generalized traveling salesman problem [6], also allows for skipping certain jobs but differs in that the jobs are partitioned into sets and at least one job from each set has to be visited by the tour.

The main results of the paper are: Facets of the one machine subproblem define facets of the m-Cost ATSP polytope for $m \geq 3$; the non-negativity constraints and the v0-cut inequalities are facet defining for the m-Cost ATSP polytope for $m \geq 2$, they can be separated in polynomial time, and they imply the subtour elimination constraints if the standard transformation for identical machines from m-ATSP to ATSP is applied. In addition, we identify a new class of so called *nested conflict inequalities* that are facet defining for the polytope of the one machine subproblem and the m-Cost ATSP polytope for $m \geq 2$.

For our real-world instances the linear relaxation based on the v0-cut inequalities yields, in average, a relative gap of 0.2% with respect to the best solution known. In comparison, the relaxation using subtour elimination constraints on the transformed problem exhibits an average relative gap of 0.5%, so this approach reduces the gap by 60%.

The paper is organized as follows. Section 2 introduces the necessary notation and gives the problem definition. In Section 3 we present an integer programming formulation of the m-Cost ATSP and determine the dimension of the m-Cost ATSP polytope. Section 4 is devoted to the one machine subproblem, its relation to the PCTSP, and the separation algorithm. In Section 5 the facet defining inequalities of the subproblem are shown to be facet defining for the m-Cost ATSP polytope. In Section 6 we prove that the v0-cut inequalities imply the subtour elimination constraints for the transformed problem. Finally, in Section 7 numerical results are presented for the scheduling problem at Herlitz PBS AG.