U-Resolution: An Inference Rule for Regular Multiple-Valued Logics*

Sonia M. Leach¹, James J. Lu², Neil V. Murray³, and Erik Rosenthal⁴

¹ Department of Computer Science, Brown University, U.S.A.
sml@cs.brown.edu
² Department of Computer Science, Bucknell University, U.S.A.
jameslu@bucknell.edu
³ Department of Computer Science, State University of New York, U.S.A.
   nvm@cs.albany.edu
⁴ Department of Mathematics, University of New Haven, U.S.A.
brodsky@charger.newhaven.edu

Abstract. The inference rule U-resolution for regular multiple-valued logics is developed. One advantage of U-resolution is that linear, regular proofs are possible. That is, unlike existing deduction techniques, U-resolution admits input deductions (for Horn sets) while maintaining regular signs. More importantly, U-resolution proofs are at least as short as proofs for definite clauses generated by the standard inference techniques—annotated resolution and reduction—and pruning of the search space occurs automatically.

1 Introduction

Signed logics [18,10] provide a general framework for reasoning about multiple-valued logics (MVL’s). They evolved from a variety of work on non-standard computational logics, including [2,3,5,6,8,15,14,16,20,22]. The key is the attachment of signs—subsets of the set of truth values—to formulas in the MVL. This approach is appealing because it facilitates the utilization of classical techniques for the analysis of non-standard logics, which reflects the essentially classical nature of human reasoning. That is, regardless of the domain of truth values associated with a logic, at the meta-level, humans interpret statements about the logic to be either true or false.

This paper focuses on the class of regular signed logics. Regular signed logics are of interest in the knowledge representation and logic programming communities because they correspond to the class of paraconsistent logics known as annotated logics, introduced by Subrahmanian [21], Blair and Subrahmanian [4], and Kifer et al. [12, 8, 25]. In [18], regular signed logics were also shown to

* This research was supported in part by the National Science Foundation under grants CCR-9731893, CCR-9404338 and CCR-9504349.

¹ Hähnle, R. and Escalada-Imaz, G. [11] have an excellent survey encompassing deductive techniques for a wide class of MVL’s, including (properly) signed logics.

© Springer-Verlag Berlin Heidelberg 1998
capture fuzzy logics, but in this paper, regular signed logics will refer to annotated logics. In particular, the focus is on the definite Horn subset of annotated logic, widely applied within logic programming. The inference rule U-resolution is developed for regular signed logics, and its relative advantages with respect to the standard inference rules for annotated logic programs (ALP’s)—annotated resolution and reduction—are described. These include the fact that linear, regular proofs are possible; furthermore, U-resolution proofs are at least as short as annotated resolution proofs, and pruning of the search space occurs automatically.

The next section is a summary of the basic ideas of signed formulas; greater detail can be found in [20] and in [18].

2 Signed Logics

We assume a language $\Lambda$ consisting of (finite) logical formulas built in the usual way from a set $\mathcal{A}$ of atoms (predicates and terms at the first order level), a set of connectives, and a set of logical constants. For the sake of completeness, we define a formula in $\Lambda$ as follows: Atoms are formulas; if $\Theta$ is an $n$-ary connective and if $F_1, F_2, \ldots, F_n$ are formulas, then so is $\Theta(F_1, F_2, \ldots, F_n)$.

Associated with $\Lambda$ is a set $\Delta$ of truth values, and an interpretation for $\Lambda$ is a function from $\mathcal{A}$ to $\Delta$; i.e., an assignment of truth values to every atom in $\Lambda$. A connective $\Theta$ of arity $n$ denotes a function $\Theta : \Delta^n \to \Delta$. Interpretations are extended in the usual way to mappings from the set of formulas to $\Delta$. Alternatively, a formula $F$ of $\Lambda$ can be regarded as denoting a mapping from interpretations to $\Delta$.

A sign $S$ is a subset of $\Delta$, and a signed formula is an expression of the form $S:F$, where $S$ is a sign and $F$ is a formula in $\Lambda$. If $F$ is an atom in $\Lambda$, we call $S:F$ a signed literal.

Signed formulas may be thought of as a formalization of meta-reasoning over MVL’s [20]. A natural interpretation of the signed formula $S:F$ is the query, “Is the truth value of $F$ in $S$?” The answer to such a query is yes or no; that is, either the formula evaluates to some element in $S$ or does not. Observe that both the query and the answer are at the meta-level; observe also that the question cannot even be formulated at the object level. On the other hand, the question, “What is the truth value of $F$?” may be interpreted at the object level since the answer is an element of $\Delta$.

For example, let $\Delta$ be the interval $[0, 1]$, where elements of $\Delta$ represent the degree of belief of some fixed reasoning agent $X$. Thus, $\{1\}:P$ can be interpreted as, “Is $X$ certain of the proposition $P$?” and $[0,1]:P$ asks, “Is $X$ quite doubtful of $P$?” These are yes or no questions.

To answer arbitrary queries, we represent queries about formulas in $\Lambda$ by formulas in a classical logic $\Lambda_S$, the language of signed formulas; it is defined as follows: The literals are signed formulas and the connectives are (classical) conjunction and disjunction. It should be emphasized that a signed formula