Some Properties of Timed Nets under the Earliest Firing Rule

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Abstract
We show that timed nets under the earliest firing rule can be transformed equivalently to Petri nets working under the maximum firing strategy. Additionally, we give two sufficient conditions for live Petri nets to remain live under any timing.

1. Definitions and Notations
Let $\mathbb{R}_0$ resp. $\mathbb{Q}_0$ denote the set of all nonnegative real resp. rational numbers. By $\mathbb{N}$ we denote the set of all nonnegative integers, and, by $\mathbb{N}^+$ the set of all positive integers.

Definition 1.
A \textit{timed net} is a tuple $TN = [P,T,F,V,m_0,D]$ where the subtuple $N = [P,T,F,V,m_0]$ is a finite Petri net (with the place set $P$, with the transition set $T$, with the arc set $F$, with the initial marking $m_0$ and, for all arcs $f$ from $F$, with the nonzero multiplicity $V(f)$) and where $D: T \rightarrow \mathbb{R}_0$ is a mapping which assigns to every transition $t$ a nonnegative real number.

For every transition $t$ the number $D(t)$ is interpreted as the duration of the firing of $t$, i.e. if $t$ fires at time $x$ the corresponding number of tokens is removed from the pre-places of $t$ at time $x$ and the corresponding number of tokens appear on the post-places of $t$ at time $x + D(t)$.

As usual, for every transition $t$ we define two markings $t^+$ and $t^-$ by
\[
t^{-}(p) := \begin{cases} 
V(p,t), & \text{if } (p,t) \in F, \\
0, & \text{else;}
\end{cases}
\]
\[
t^{+}(p) := \begin{cases} 
V(t,p), & \text{if } (t,p) \in F, \\
0, & \text{else.}
\end{cases}
\]

A transition \( t \) has concession at the marking \( m \) in TN iff it has concession at \( m \) in \( N \), i.e. iff \( t^{-} \leq m \).

We assume here the so-called earliest firing schedule, i.e. a transition obtaining concession (by the change of the marking) at time \( x \) is obliged to fire at this time (if there is no conflict).

If in TN there exist transitions \( t \) with \( D(t) = 0 \) then, even in the absence of conflicts, the problem of determining the marking at some time \( x > 0 \) (the marking at time \( x = 0 \) is \( m_0 \)) arises, because such a transition can change the marking without consuming time.

In general the assumption is made that 'timeless' transitions fire first as long as there are some having concession. Then one has to assume that this procedure comes to an end in any case, which leads to additional assumptions on the behaviour and the structure of the net, e.g. that there exists no realizable \( T \)-invariant such that only timeless transitions are involved.

In this paper we restrict ourselves to timed nets where

\[ D(t) > 0 \text{ for all } t \text{ from } T, \]

moreover, we assume the values \( D(t) \) to be rational:

\[ D : T \to \mathbb{Q}. \]

Since \( T \) is finite, the last restriction enables us to stretch the time scale in such a way that within the new time scale all the durations are positive natural numbers (by multiplication with the least common multiple of the denominators of all the quotients \( D(t) \)) without changing the structure of the reachability set, and, therefore, without changing such properties as boundedness, liveness etc..

We resume: