NUMERICAL SIMULATION OF TRANSITION TO TURBULENCE USING HIGHER ORDER METHOD OF LINES

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1 Introduction

The pseudo-spectral method is accurate and efficient so that it is widely used. The transition to the turbulence and large eddy simulations of turbulent flows are treated by the spectral method [1-2]. In calculating turbulent flows with finite difference method, however, there exist a number of difficulties although these methods are widely used in the computational fluid dynamics. The high accuracy and high efficiency have to be assured. The formulation is also important in calculating three-dimensional incompressible flows. As is well known, it is difficult to satisfy the solenoidal condition for the velocity field in the primitive variable formulation. From these points of views we propose a new computational method, for calculating turbulent shear flows, which is composed of the higher order method of lines combined with the vorticity-vector potential formulation. Although the solenoidal condition is satisfied automatically, the Poisson equations have to be solved for three components of the vector potential. The multi-grid method is therefore used in order to accelerate the convergence of the Poisson equations. The computations are carried out with the super computer Fujitsu FACOM VP-400E at the Data Processing Center of Kyoto University.

2 Governing Equations and Computational Methods

In two-dimensional flows the vorticity and the stream function formulation is often used. The vector potential $\psi$, however, introduced on three-dimensional flows from $u = \text{rot} \psi$ where $u$ denotes the velocity field. The Navier-Stokes equations are then transformed into the vorticity transport equations for the vorticity $\zeta = \text{rot} u$ and the Poisson equations are defined for the vector potential $\psi$. In the present formulation the equation of continuity is automatically satisfied and the present method is expected to be an efficient computational method for solving incompressible flows.

\[
\frac{\partial \zeta_x}{\partial t} = - \nabla \cdot (u \zeta_x) + (\zeta \cdot \nabla) u + \frac{1}{Re} \nabla^2 \zeta_x, \quad (1)
\]

\[
\frac{\partial \zeta_y}{\partial t} = - \nabla \cdot (u \zeta_y) + (\zeta \cdot \nabla) v + \frac{1}{Re} \nabla^2 \zeta_y, \quad (2)
\]

\[
\frac{\partial \zeta_z}{\partial t} = - \nabla \cdot (u \zeta_z) + (\zeta \cdot \nabla) w + \frac{1}{Re} \nabla^2 \zeta_z, \quad (3)
\]

\[
\nabla^2 \psi = -\zeta. \quad (4)
\]

The method of lines is adopted in the present investigation. In this method spatial discretizations and the time integration are treated separately. For spatial discretizations the eighth order modified differential quadrature (MDQ) method is used. Partial derivatives of $\zeta$ for the $x$-direction, for instance, are approximated in terms of vorticity values $\zeta_{ijk} = \zeta(x_i, y_j, z_k)$ at the grid point $(x_i, y_j, z_k)$, according to the following expressions:

\[
\frac{\partial \zeta}{\partial x_{ijk}} = \sum_{l=-4}^{4} a_l \zeta_{i+l,j,k}, \quad (5)
\]

\[
\frac{\partial^2 \zeta}{\partial x^2_{ijk}} = \sum_{l=-4}^{4} b_l \zeta_{i+l,j,k}, \quad b_u = \sum_{l=-4}^{4} a_{im} a_{ml}, \quad (6)
\]
where \( a_{ij} \) are numerical coefficients specified adequately [3]. The same discretizations are yielded for derivatives in the \( y- \) and \( z- \) direction. After spatial discretizations, the partial differential equations (PDEs) are reduced to a set of ordinary differential equations (ODEs) for the vorticity values at all inner grid points.

\[
\frac{d\zeta}{dt} = \mathbf{F}(\zeta), \tag{7}
\]

\[
\zeta = (\zeta_{x,2,2}, \zeta_{x,3,2}, \ldots, \zeta_{x,imax-1,1,kmaz-1})^T. \tag{8}
\]

In expressions (8) \( imax \), \( jmax \) and \( kmaz \) represent the number of grid points in the \( x- \), \( y- \) and \( z- \) direction, respectively. The fourth step Runge-Kutta-Gill Method is used for the time integration of ODEs (7) with (8). The Poisson equations are discretized by the MDQ method as (4) and the higher order multi-grid method is used for the relaxation.

### 3 Computational Results

The present computational method has been already validated for the steady and unsteady flow [4]. In the present paper, therefore, the numerical simulation of the subcritical transition to the turbulence in a plane channel is treated with the eighth accuracy. The computational domain is shown in Fig. 1. The initial condition is composed of the Benney-Lin type flow [3]. The computation is carried out at \( Re = 1500 \) and in the number of grid points 65 \( \times \) 65 \( \times \) 129. The Figure 2 shows the spanwise vorticity contour on the plane \( z = 0 \) at \( t = 21 \). The horseshoe vortices are elongated, bent extremely and touch another newly generated vortices. This is the perspective view of the transient turbulent shear flows. In Fig. 3 we show constant enstropy surfaces with four levels. The structure of the horseshoe vortex is clearly visible. The three-dimensional vortices are extremely bent in both the spanwise direction and the normal direction to the wall. In order to inspect feasibility of the direct numerical simulation (DNS) of the turbulent shear flow, the energy spectrum on the \( x - z \) plane are shown in Fig. 4. At time \( t = 18.75 \), the energy spectrum does not attain the maximum wave number region. At \( t = 21 \), however, the energy spectrum covers all wave number regions, so that DNS is not feasible after this time.

### 4 Conclusions

The higher order method of lines approach proposed here has several important advantages to calculate shear flow turbulence directly.

(1) The present method adopts the vector potential method and satisfies the divergent-free condition for the velocity field automatically so that the spurious error is excluded.

(2) The eighth order accurate method is used, and therefore numerical dissipations is eliminated completely.

(3) The transition process of the shear flow turbulence is captured till the time 21 in the 65 \( \times \) 65 \( \times \) 129 grid.

(4) The computer graphics displays show that the transient turbulent shear flow is composed of extremely elongated and rolled up horseshoe vortices.

### References