The Flow Computation of a Liquid Rocket Engine Combustor of Complex Geometry

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Abstract

A computational model for the gasphase flow in arbitrarily-shaped combustors is formulated and applied to the flow computation of a liquid rocket engine combustor with an acoustic cavity. The SIMPLER algorithm with staggered grids is revised for a general non-orthogonal curvilinear coordinate system. The contravariant velocity components are selected as the dependent variables of the transformed momentum equations. However, the usual nine-point formulation for the pressure equation is avoided in a two-dimensional flow field by some manipulations of the transformed momentum equations of Cartesian velocity components. This computational model has been tested satisfactorily by the comparison of the numerical solutions with the experimental results of a laminar flow through a tube with an axisymmetric contraction. The flow computation of a liquid rocket engine combustor with a hypothetic acoustic cavity is then conducted to study the detailed flow pattern around the acoustic cavity. The strongly recirculating flow in and out of the acoustic cavity has been predicted, and the effects of combustion on the flow pattern have been examined.

Introduction

The modern design of aerospace propulsion system calls for the sophisticated computer code to aid in the analysis of combustion performance of combustors of complex geometry such as the variable thrust engine (VTE) in the orbit maneuvering vehicle (OMV) [1]. However, before all the physical models can be implemented in the computer code to describe the complex heterogeneous combustion phenomena, the accurate flow computation is considered as the basis and should be assured beforehand. Among all the computational models installed in the computer code for the flow computation, the computational algorithm to compute the gasphase flow in the general body-fitted non-orthogonal curvilinear coordinates is obviously the most essential part of the whole computer code. It deserves more attention than others, and its accuracy and efficiency should be examined before the code development can be further pursued.

The central issue to solve the flow field in a non-orthogonal curvilinear coordinates is the proper selection of the dependent variables of the transformed momentum equations. The possible candidates for these dependent variables include Cartesian velocity components [2], covariant velocity components [3] and contravariant velocity components [4]. The selection of Cartesian velocity components has the merit of simpler transformed equations. However, it is needed to solve all the Cartesian velocity components at each control-volume face to obtain the contravariant velocity components which are used to interpret the convective flow through the control-volume faces. This drawback implies that more computational efforts are needed if the staggered grid system is used. The selection of covariant velocity components as the dependent variables of the transformed momentum equations warrants the five-point formulation in the pressure and pressure correction equations in a two-dimensional flow situation. This is because of the alignment of covariant velocity components with the grid lines such that only the pressure gradient's component along the grid line exists in the transformed momentum equations. However, in order to obtain the contravariant velocity components to describe the convective flow through the control-volume faces, interpolation is usually performed to acquire the other covariant velocity components so that the contravariant
velocity components at the control-volume faces can be estimated. Furthermore, the continuity
equation in terms of the covariant velocity components is generally nonhomogenous due to the non-
orthogonal grids. On the other hand, the selection of contravariant velocity components as the
dependent variables of the transformed momentum equations has the following advantages. First,
the transformed continuity equation, which is used to formulate both pressure equation and pres-
sure correction equation in the SIMPLER algorithm [5], preserves the same equation form exactly
as that in the orthogonal coordinate system, and does not possess a nonhomogeneous source term
for the non-orthogonal grids. Therefore, the same solution procedures for the pressure equation
and the pressure correction equation as those in the orthogonal coordinate system can be used.
Secondly, since the contravariant velocity components are computed directly from the transformed
momentum equations, it is not necessary to exercise the interpolation or to compute all the velocity
components to obtain the contravariant velocity components at every control-volume face.
The difficulty associated with the selection of contravariant velocity components as the depen-
dent variables of the transformed momentum equations is that the direction of pressure gradient
is not aligned with the contravariant velocity components in the non-orthogonal grids in general.
Therefore, a nine-point formulation for the pressure equation in a two-dimensional flow comput-
tation was usually proposed to overcome this difficulty. The present paper, however, treats this
problem in a different way. The transformed continuity equation and the transformed momentum
equations are arranged to precisely preserve the same forms as those in the orthogonal curvilinear
coordinates. Therefore, a five-point formulation can be proposed, and the revision of SIMPLER
algorithm for the non-orthogonal curvilinear coordinates is straightforward and much simpler than
others.

Governing Equations

The steady conservation equations for a general dependent variable $\phi$ can be given in the
following general form in terms of the non-orthogonal curvilinear coordinates $(\xi, \eta)$:
\[
\frac{1}{\sqrt{g}} \frac{\partial}{\partial \xi} \left( \sqrt{g} \rho U \phi - \sqrt{g} g^{11} \Gamma_{\phi} \frac{\partial \phi}{\partial \xi} \right) + \frac{1}{\sqrt{g}} \frac{\partial}{\partial \eta} \left( \sqrt{g} \rho V \phi - \sqrt{g} g^{22} \Gamma_{\phi} \frac{\partial \phi}{\partial \eta} \right) = S
\] (1)
This general form can also represent the continuity equation and the momentum equations as $\phi$
is taken to be 1 and the Cartesian velocity components respectively. For the turbulent flow, K-$\varepsilon$
two-equation turbulence model is used in which wall function method is adopted in the near wall
region. The combustion process is assumed to be controlled by both chemical kinetics and turbulent
mixing, therefore, the reaction rate is dominated by the slower one. The one-step Arrhenius rate
law is adopted for the process controlled by the chemical kinetics, while the EBU (Eddy-Break-Up)
model [6] is used to estimate the rate of turbulent mixing. Namely, the reaction rate is given by
\[
\omega = - \min (|| R_{Arr} ||, || R_{EBU} ||).
\] (2)

Computational Algorithm

From the transformed momentum equations with Cartesian velocity components being the
dependent variables, the transformed momentum equation associated with contravariant velocity
component $U$ can be derived from $u$-equation as:
\[
\frac{1}{\sqrt{g}} \frac{\partial}{\partial \xi} \left( \sqrt{g} \rho U x_{\xi} U - \sqrt{g} g^{11} \mu_{eff} \frac{\partial x_{\xi} U}{\partial \xi} \right) + \frac{1}{\sqrt{g}} \frac{\partial}{\partial \eta} \left( \sqrt{g} \rho V x_{\xi} U - \sqrt{g} g^{22} \mu_{eff} \frac{\partial x_{\xi} U}{\partial \eta} \right)
= - \frac{1}{\sqrt{g}} \left( y_{\eta} \frac{\partial p}{\partial \xi} - y_{\xi} \frac{\partial p}{\partial \eta} \right) + S \nu
\] (3)
or derived from $v$-equation as:
\[
\frac{1}{\sqrt{g}} \frac{\partial}{\partial \xi} \left( \sqrt{g} \rho U y_{\xi} U - \sqrt{g} g^{11} \mu_{eff} \frac{\partial y_{\xi} U}{\partial \xi} \right) + \frac{1}{\sqrt{g}} \frac{\partial}{\partial \eta} \left( \sqrt{g} \rho V y_{\xi} U - \sqrt{g} g^{22} \mu_{eff} \frac{\partial y_{\xi} U}{\partial \eta} \right)
= - \frac{1}{\sqrt{g}} \left( x_{\eta} \frac{\partial p}{\partial \eta} - x_{\xi} \frac{\partial p}{\partial \xi} \right) + S \nu
\] (4)