Quantum groups, Riemann surfaces and conformal field theory

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Abstract: An explicit representation is constructed, starting with the operator algebra corresponding to the Coulomb gas representation of conformal field theories. By "quantizing" the uniformization theory of Riemann surfaces, the geometric interpretation of such a representation is obtained.

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1. Introduction: Quantum Groups and Ice-Type Models

Historically, quantum groups appear as the most natural mathematical framework to use in the construction of solutions to the quantum Yang-Baxter equation [1, 2, 3]. More precisely quantum groups provide a way to construct generalized ice-type models [4].

To be more concrete let us consider the simple example of Baxter's six-vertex model. The lattice variables in this case can take only two values $\pm 1$ with the Boltzmann weights satisfying the Yang-Baxter equation:

$$
\sum_{\mu'\nu'\gamma} W(\mu_\alpha \mu_\beta | u) W(\nu_\gamma \nu_\delta | u + v) W(\nu_\delta \nu_\gamma | v) = 0
$$

(1)

An important type of solutions to (1) are the trigonometric ones:

$$
W(\mu \nu \rho \delta | u) = \delta_{ac} + \delta_{bd} \left( \frac{\sin u}{\sin (\lambda - u)} \right) e^{-i(\alpha + c - 2b)/2}
$$

(2)

where we have used the "IRF-variables" $a, b, \ldots$ living on the semiinfinite Coxeter graph of type A:

$$
\begin{array}{cccccc}
0 & 1 & 2 & 3 & \rightarrow \infty
\end{array}
$$

(3)

and the vertex variables $\mu, \nu, \ldots$ are defined according to the vertex-IRF Kadanoff-Wegner map [5]:

$$
\begin{array}{ccc}
\gamma & \mu & \delta \\
\mu & \rho & \delta \\
\delta & \rho & \gamma
\end{array}
$$

(4)

In the limit $u \rightarrow \infty$ this trigonometric solution defines the R-matrix of $SU(2)_q$:

$$
R = \begin{pmatrix}
q & 1 & q^{-1} \\
1 & q^{-1} & q
\end{pmatrix}
$$

(5)

where $q$ depends on the free parameter $\lambda$ appearing in (2). The quantum group $SU(2)_q$ is defined by:

$$
[X^\pm, H] = \mp 2X^\pm
$$

$$
[X^+, X^-] = [H]
$$

(6)

with the $q$-number $[a]$ given by $(q^a - q^{-a})/(q - q^{-1})$. The comultiplication compatible, as an homomorphism, with the commutation relations (6) is:

$$
\Delta(X^\pm) = X^\pm \otimes q^{H/2} + q^{-H/2} \otimes X^\pm
$$

$$
\Delta(H) = H \otimes 1 + 1 \otimes H
$$

(7)