Petri Net Models of a Distributed Election Protocol on a Unidirectional Ring

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ABSTRACT This paper is devoted to the performance analysis of an election protocol for a unidirectional ring proposed by C.Kaiser. We give two models of increasing complexity in order to compute several performance criteria. The two models (called initiation model and evaluation model) are defined using coloured Petri nets. For these models several qualitative properties are proven. Then the required performance criteria are computed using an original computation method defined for non repetitive stochastic Petri net.

Keywords: distributed algorithms, election protocols, formal proof, performance evaluation, stochastic Petri nets.

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INTRODUCTION

An election protocol on a unidirectional ring of processors [GAR 81, LEL 77, RAY 85] is a distributed algorithm that elects a unique processor as a leader. This selected processor acts, for instance, as a distributed protocol coordinator.

This problem has been stated by Lelann [LEL 77] for recovering from token failure. Chang and Roberts [CHA 79] have proposed the idea of message extinction which is the underlying principle of all the classical algorithms for unidirectional as well as for bidirectional rings. The Chang-Roberts protocol leads to a traffic in $O(n \cdot \log n)$ elementary messages and to a response time in $O(3n)$.

This paper deals with the analysis of a distributed algorithm proposed by C. Kaiser [KAI 89]. The discussion that follows and another analytic study [JEA 88] show that this algorithm leads to a higher traffic but to a lower response time than the classical algorithms. The latter performance aspect may be of particular interest for real-time applications.

The first section of the paper is devoted to an informal presentation of the algorithm. Although our focus is indeed on analytic techniques, the methodology presented here starts from coloured Petri nets models [JEN 81]. These models are designed to prove some properties of the algorithm but mainly to compute several performance criteria using the stochastic Petri net and coloured stochastic Petri net method [FLO 85] [ZEN 85]. So in the second section we present two coloured stochastic Petri nets models of the election algorithm. These models have an increasing complexity and are designed to apply to the evaluation of specific criteria with the minimum expansion of the state space. The last section is devoted to the formal proofs and to the presentation of numerical results.

I ELECTION ON A UNIDIRECTIONAL RING

I.1 Hypothesis

We consider a set of $n$ processors linked by a ring network. We number processors according to their relative positions on the ring (processor $P_i$ is linked to its predecessor $P_{i-1 \mod(n)}$ and its