Ultra-relativistic Pulsar Wind

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Abstract: We show that cold, ultra-relativistic hydromagnetic winds can have a kinetic energy flux much larger than the Poynting flux. Even slight deviation of streamlines from a perfect radial geometry can move the fast magnetosonic point to some finite radii comparable to that of the light cylinder. Once the flow becomes supermagnetosonic, divergence of the streamlines can accelerate it via the “magnetic nozzle” effect, thereby converting Poyting flux to kinetic energy. We discuss our results in the content of pulsar winds.

1 Introduction

Shortly after the discovery of pulsars, it was proposed that a significant portion of the spindown energy could go into a relativistic wind propelled by the “magnetic slingshot” effect. By considering a relativistic ($\mu \gg 1, \mu c^2$: specific total energy) radial cold wind operating under this mechanisim, Michel (1969) showed that the Poynting flux always dominates the kinetic energy in the ratio $\mu : \mu^{2/3}$. However, models for the interior of the Crab nebula (Rees and Gunn 1974; Kennel and Coroniti 1984) require that the kinetic energy flux in the pulsar wind be several hundred times larger than the Poynting flux when the wind reaches the standing shock. This forced Kennel and Coroniti (1984) to conclude that the wind could therefore not be driven primarily by the magnetic slingshot effect, but rather must be driven thermally by the pressure of relativistically hot plasma.

Large ratio of kinetic energy to Poynting flux, however, is possible when the flow is collimated. In this case, slight deviation from a perfect radial wind geometry can move the fast magnetosonic point, which is always at infinity for a radial cold wind (Goldreich and Julian 1970), rapidly toward the light cylinder. Further acceleration of the flow beyond the fast point is obtained through a “magnetic nozzle” effect, provided that the streamlines diverge sufficiently.
2 Location of the Fast Magnetosonic Point

For a axisymmetric, steady-state, relativistic cold MHD wind, the conservation of the total specific energy $\mu c^2$ and angular momentum $l$, when combined with the flux frozen condition, yields an energy equation:

$$
\left( \mu - \frac{x_A^2 - \mu x^2}{1 - x^2(1 + \tau)} \right)^2 = 1 + (a\tau)^2 + \frac{x_A^4}{x^2} \left( 1 - \frac{1 - \mu(x/x_A)^2}{1 - x^2(1 + \tau)} \right)^2
$$

where

$$
x \equiv \frac{R\Omega}{c}, \quad x_A \equiv \sqrt{\frac{l\Omega}{c^2}}, \quad a \equiv \frac{B_p R^2}{k c^3/\Omega^2}, \quad \tau \equiv \frac{k^2}{4\pi \rho z^2}
$$

and the mass flux per unit magnetic flux $k$ and the angular velocity $\Omega$ are constant along a flux surface, while other quantities have their standard meanings. If the flow starts from $x=0$ at rest, then $x_A = \sqrt{\mu - 1}$.

The flow poloidal 4-speed $u_p(= \omega r)$ is completely determined by the local value of the quantity $a$ through eq. (1). In case of a radial wind, $a$ is just the Michel’s (1969) magnetization parameter $\sigma$, which does not change throughout the flow. For collimated flows, the deviation from a perfect radial wind is measured by a dimensionless parameter $\delta(x) = d \ln a/d \ln x$.

Making use of the fact that, at the fast point, two branches of root of eq. (1) cross, one can relate the critical energy $\mu_c$, the deviation $\delta_f$ and the kinetic energy $\gamma_f$ at the fast point to the location of the fast point $x_f$. The results are plotted in Fig. (1), from which we conclude:

1. Small deviation $\delta$ can indeed move the fast point to a few light cylinder radii.
2. Critical total specific energy $\mu_c$ is close to the value of $a$ at the fast point $a_f$.
3. The Poynting flux is still much larger than the kinetic energy up to the fast point, and further acceleration beyond the fast point is needed to have a kinetically dominated flow.

3 Acceleration Beyond the Fast Point

Well beyond the light cylinder ($x \gg 1$), the energy equation is simplified to:

$$
\left( \frac{\tau}{1 + \tau} \right)^2 = 1 + (a\tau)^2.
$$

If we denote the ratio of the Poynting flux to the kinetic energy by $\sigma$, then $\sigma = 1/\tau$. According to the above equation, an equipartition ($\sigma = 1$) is reached when $a \approx \mu/2 \approx a_f/2$. To get even smaller $\sigma$, a small $a(< \mu \gg 1)$ is required. In this limit, two branches of solution to eq. (1a) exist, namely:

$$
\sigma \approx a/\mu \approx a_f/\mu \ll 1
$$

and