Abstract. We study multipacket routing problems. We divide the multipacket routing problem into two classes, namely, distance limited and bisection limited routing problems. Then, we concentrate on rings of processors. Having a full understanding of the multipacket routing problem on rings is essential before trying to attack the problem for the more general case of r-dimensional meshes and tori. We prove a new lower bound of \( \frac{2n}{3} \) routing steps for the case of distance limited routing problems. We also give an algorithm that tightens this lower bound. For bisection limited problems, we present an algorithm that completes the routing in near optimal time.

1 Introduction

A great deal of work has been devoted to the study of the packet routing problem [1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12]. This is because the packet routing problem is closely related to parallel computation. Through the routing of messages (packets) we are able to emulate shared memory [11]. More generally, for a parallel computer to be computationally effective, it must be able to route messages from their origin processors to their destination processors fast and with small, preferably constant size queues. These queues are created while two or more packets are waiting to cross the same communication channel.

In this paper, we consider two types of packet routing problems, namely, distance limited and bisection limited routing problems, a distinction which is based on the number of packets each processor has to route. We concentrate on permutation problems on a ring of processors. The reason for doing so, is because, before trying to attack the problem for the more general case of r-dimensional meshes and tori, we must have a full understanding of the problem in its simplest form. We prove a new lower bound for distance limited problems on rings and we give an algorithm that matches the lower bound. For the case of bisection limited routing problems, we present an algorithm that completes the routing in near optimal time.
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A ring of processors is defined to be a graph $G = (V, E)$ where, $V = \{i | i = 0, 1, 2, ..., n - 1\}$ and an edge $e = (i, j)$ belongs to $E$ if $|j - i| = 1$ or $|j - i| = n - 1$. At any step, each processor can communicate with both of its neighbors.

We define the distance along the shortest path between processors $P_1 = i$ and $P_2 = j$, denoted $D_s(P_1, P_2)$, to be the minimum number of links that a packet has to traverse starting from processor $P_1$ and destined for processor $P_2$. Obviously, $D_s(P_1, P_2) = D_s(P_2, P_1)$. Formally, for processors $P_1 = i$ and $P_2 = j$, $j > i$, we define $D_s(P_1, P_2) = \min\{(j - i), n - (j - i)\}$.

In a permutation routing problem each processor has one packet to transmit to any other processor. At the end, each processor receives exactly one packet. In the multipacket permutation problem each processor has $k$ packets all of which are destined for the same processor. At the end, each processor receives exactly $k$ packets. This problem arises when a single packet in the permutation routing problem consists of $k$ flits. Some work has already been done on square meshes for this case: Simvonis and Makedon [8] treated the $k$ flits as an unbreakable "snake", while Kunde and Tensi [4] routed the flits of a packet independently. Up to now, however, no work has been done on rings.

The remainder of the paper is organized into sections as follows. In Section 2, we define the classes of distance limited and bisection limited routing problems. In Section 3, we concentrate on distance limited problems on rings of $n$ processors. We present a lower bound of $\frac{2n}{3}$ steps, and we give an algorithm that matches that bound. In Section 4, we investigate the bisection limited problem on a ring of $n$ processors. The known lower bound for this problem is $\frac{kn}{4}$ routing steps. We present an algorithm that routes any problem in at most $\frac{kn}{4} + \frac{5n}{2}$ routing steps. Finally, in Section 5, we discuss further work that has to be done in this area.

2 Two Types of Routing Problems

We obtain lower bounds on the number of steps required to solve a routing problem using two different arguments. The first one is a lower bound based on the maximum distance a packet has to travel (distance bound). The second one is based on the bisection bound of the network used. Then, the lower bound is $\text{max}(\text{distance bound, bisection bound})$.

For the case of $r$-dimensional meshes of side-length $n$, the distance bound is $r(n - 1)$ and the bisection bound is $\frac{n k}{2}$, where $k$ is the number of packets each processor holds. Thus, the lower bound on the number of steps required to solve the multipacket permutation routing problem on the $r$-dimensional mesh is $\text{max}\{r(n - 1), \frac{n k}{2}\}$. Similarly, for the torus, the lower bound is $\text{max}\{\frac{r(n-1)}{2}, \frac{n k}{4}\}$.

It is clear from the above that we can divide the routing problems into two categories: the bisection-limited problems and the distance-limited problems. A problem is bisection-limited if the bisection lower bound is greater than the distance lower bound. Otherwise,