ABDUCTIVE SYSTEMS FOR NON-MONOTONIC REASONING

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1. Introduction

This paper presents a new universal framework for formalizing Non-Monotonic reasoning. The notion of Abductive System is a generalization of the notion of Abductive Framework, proposed in [2]. In comparison with existing universal approaches to Non-Monotonic reasoning (such as the Logic of Grounded Knowledge [3], or Non-monotonic Rule Systems [4]), the formalism, presented here has some advantages:

- it combines preferential semantics with fixed-point approach;
- it is simple;
- it is general enough.

In this paper it is shown, that Default Logic [6], Autoepistemic Logic [5], the Logic of Grounded Knowledge [3], Non-Monotonic Modal Logic [7], and Stable Model Semantics for Logic Programming [1] can be expressed in terms of Abductive Systems.

The paper also presents two new Non-Monotonic logics:

- the Logic of Knowledge and Beliefs (KBL);
- Bimodal Non-Monotonic Logic (BNL).

KBL is a formalization of reasoning of a person about his own knowledge and beliefs. BNL is a modification of the Logic of Grounded Knowledge [3], and has some advantages in comparison with it.

2. Abductive systems

Definition 1: An abductive system is a quadruple \( \langle L, \vdash, D, C \rangle \) where

- \( L \) is a formal language with negation "\( \sim \)"
- \( \vdash \subseteq 2^L \times L \) is a provability relation, defined by a deductive system;
- \( D \subseteq L \) is an abductive set;
- \( C \subseteq 2^L \) is a set of allowed extensions.

Let \( T \subseteq L \); \( \varphi_1, \ldots, \varphi_n \in L \). Introduce abbreviations:

1) \( T \vdash \perp \) iff there is \( \varphi \in L \), such that \( T \vdash \varphi \) and \( T \vdash \sim \varphi \).
In this case $T$ is called inconsistent.

2) $T \not \vdash \bot$, iff it isn't true that $T \vdash \bot$.

In this case $T$ is called consistent.

3) $T, \varphi_1, \ldots, \varphi_n \not \vdash \psi$ iff $T \cup \{\varphi_1, \ldots, \varphi_n\} \not \vdash \psi$.

4) $\text{Th}(T) = \{ \varphi \mid T \vdash \varphi \}$.

Fix an abductive system $\Sigma = \langle L, \vdash, D, C \rangle$.

**Definition 2:** Let $E, T \subseteq L$. Then $E$ is an extension of $T$ in $\Sigma$, iff

1) $E = \text{Th}(T \cup \{\varphi \mid E, \varphi \not \vdash, \varphi \not \in D\})$; and
2) $E \in C$.

A set of formulas $E$ is called a maximal set for $T$ in $\Sigma$, iff $E$ is a maximal consistent set of the form $\text{Th}(T \cup Q)$, where $Q \subseteq D$.

**Theorem 1.** Let $T, E \subseteq L, TV \subseteq L$. Then $E$ is an extension of $T$ in $\Sigma$, iff

1) $E$ is a maximal set for $T$ in $\Sigma$; and
2) $E \in C$.

The proof of this theorem is obvious.

Introduce abbreviations:

1) $\text{Th}^1 \varphi$, iff either there is an extension of $T$, including $\varphi$, or $T$ has no extensions.
2) $\text{Th}^2 \varphi$, iff $\varphi$ is a member of all extensions of $T$.

**Definition 3:** A quadruple $\langle M, \ll, \ll, Q \rangle$ is a semantical structure for a formal language $L$ iff $M$ is a nonempty set; $\ll \subseteq M \times L$; $\ll \subseteq M \times M$ is a reflexive and transitive relation on $M$; and $Q \subseteq 2^M$.

Let $T \subseteq L$; $\varphi \in L$; $m, m_1, m_2 \in M$; $H \subseteq M$. Introduce abbreviations:

1) $m \not \models \varphi$ iff it isn't true that $m \models \varphi$;
2) $m \not \models T$ iff for any $\varphi \in T$, $m \not \models \varphi$ holds;
3) $T \not \models \varphi$ iff for any $m \in M$: $m \models T$ imply $m \models \varphi$;
4) $\text{D}(m) = \{ \varphi \mid m \not \models \varphi, \varphi \in D \}$;
5) $H \not \models T$ iff for any $m \in H$, $m \not \models T$ holds;
6) $\text{Mod}(T) = \{ m \mid m \models T \}$.
7) $m_1 \ll m_2$ iff $m_1 \ll m_2$ and $m_2 \not \ll m_1$;
8) $m_1 < m_2$ iff $m_1 \ll m_2$ and not $m_1 \ll m_2$;
9) $e(m, T) = \{ m' \mid m \models^* m', m' \models T \}$.

Let $S = \langle M, \ll, \ll, Q \rangle$ be a semantical structure for $L$. 