The Complexity of Approximating PSPACE-Complete Problems for Hierarchical Specifications
(Extended Abstract)

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Abstract

We extend the concept of polynomial time approximation algorithms to apply to problems for hierarchically specified graphs, many of which are PSPACE-complete. Assuming P \neq PSPACE, the existence or nonexistence of such efficient approximation algorithms is characterized, for several standard graph theoretic and combinatorial problems. We present polynomial time approximation algorithms for several standard problems considered in the literature. In contrast, we show that unless P = PSPACE, there is no polynomial time \( \epsilon \)-approximation for any \( \epsilon > 0 \), for several other problems, when the instances are specified hierarchically.

1 Introduction

Hierarchical system design is becoming increasingly important with the development of VLSI technology. At present, many VLSI circuits already have over a million transistors. (For example, the Intel i860 chip has about 2.5 million transistors.) Although VLSI circuits can have millions of transistors, they usually have highly regular structures. These regular structures often make them amenable to hierarchical design, specification and analysis.

Over the last decade several theoretical models have been put forward to succinctly represent objects hierarchically [2, 4, 14, 18]. Here, we use the model defined in Lengauer [6, 11, 14, 15] to describe hierarchically specified graphs. Using this model, Lengauer et al. [12, 13, 14] have given efficient algorithms to solve several graph theoretic problems including minimum spanning forests, planarity testing etc.

Here we extend the concept of polynomial time approximation algorithms so as to apply to problems for hierarchically specified graphs including PSPACE-complete such problems. We characterize the existence or nonexistence (assuming P \neq PSPACE) of polynomial time approximation algorithms, for several standard graph problems. Both positive and negative results are obtained (see

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Tables 1 and 2). Our study of approximation algorithms for hierarchically specified problems is motivated by the following two facts:

1. $\Theta(n)$ size hierarchical specifications can specify $2^{\Omega(n)}$ size graphs.

2. Many basic graph theoretic properties are PSPACE-complete [7, 11], rather than NP-complete.

For these reasons the known approximation algorithms in the literature are not applicable to graph problems, when graphs are specified hierarchically.

What we mean by a polynomial time approximation algorithm for a graph problem, when the graph is specified hierarchically, can be best understood by means of an example.

**Example:** Consider the minimum vertex cover problem, where the input is a hierarchical specification of a graph $G$. We wish to compute the size of a minimum vertex cover of $G$. Our polynomial time approximation algorithm for the vertex cover problem computes the size of an approximate vertex cover and runs in time polynomial in the size of the hierarchical description, rather than the size of $G$. Moreover, it also solves in polynomial time (in the size of the hierarchical specification) the following query problem: Given any vertex $v$ of $G$ and the path from the root to the node in the hierarchy tree in which $v$ occurs, determine whether $v$ belongs to the approximate vertex cover so computed.

This is a natural extension of the definition of approximation algorithms in the flat (i.e. non-hierarchical) case. This can be seen as follows: In the flat case, the number of vertices is polynomial in the size of the description. Given this, any polynomial time algorithm to determine if a vertex $v$ of $G$ is in the approximate minimum vertex cover can be modified easily into a polynomial time algorithm that lists all the vertices of $G$ in the approximate minimum vertex cover. For an optimization problem or a query problem, our algorithms use space and time which is a low level polynomial in the size of the hierarchical specification and thus $O(poly \log N)$ in the size of the specified graph, when the size $N$ of the graph is exponential in the size of the specification. Moreover, when we need to output the subset of vertices, subset of edges, etc. corresponding to a vertex cover, maximal matching, etc., in the expanded graph, our algorithms take essentially the same time but substantially less (often exponentially less) space than algorithms that work directly on the expanded graph.

We believe that this is the first time efficient approximation algorithms with good performance guarantees have been provided both for hierarchically specified graph problems and for PSPACE-complete problems. Thus by providing algorithms which exploit the underlying structure, we extend the range of applicability of standard algorithms. Our results are summarized as follows.

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\[2^\text{independently, Condon et al. [3] have investigated the approximability of other PSPACE-complete problems.}\]