A Universal Turing Machine

Stål Aanderaa

University of Oslo
staal.aanderaa@math.uio.no

Abstract. The aim of this paper is to give an example of a universal Turing machine, which is somewhat small. To get a small universal Turing machine a common constructions would go through simulating tag system (see Minsky 1967). The universal machine here simulate two-symbol Turing machines directly.

The Turing machine is defined by the Figure 1 or the Table 1. Suppose the universal Turing machine should simulate the Turing machine defined by the Figure 2 or by the Table 2, starting by the instantaneous description

\[ 010p_0100 \]  \hspace{1cm} (1)

Then the universal Turing machine UTm should start by the instantaneous description

\[ 01AA\bar{b}_0baac^3d^{599}c^d^{14}c^3d^{708}c^d^{15}c^3d^{599}c^d^{10}c^3d^{644}c^d^{11}c. \]  \hspace{1cm} (2)

Here the first three symbols in (2): 01A, code the first three symbols in (1): 010. The next two symbols in (2): \( AB \) code the state symbol \( p_0 \) in (1). \( q_0 \) in (2) denote the state of the universal Turing machine. The symbols \( baa \) in (2) code the last three symbols 100 in (1). The last part of (2):

\[ c^3d^{599}c^d^{14}c^3d^{708}c^d^{15}c^3d^{599}c^d^{10}c^3d^{644}c^d^{11}c. \]  \hspace{1cm} (3)

codes the Turing machine \( T_{\text{m}} \) defined in Table 2. The exponents are calculated as follows:

\[
\begin{array}{cccccccc}
\text{A} & \text{B} & \text{C} & \text{D} & \text{E} & \text{F} \\
\text{(i)} & 0p_00 \rightarrow 0H0 & AABA & (0011)_2 = 2 & RAAA0, (21113)_4 = 599 \\
\text{(ii)} & 0p_01 \rightarrow L p_101 & AAB\bar{B} & (0011)_2 = 3 & LBB01, (00032)_4 = 14 \\
\text{(iii)} & 0p_10 \rightarrow R 0p_0 & AB\bar{B}_A & (0111)_2 = 6 & R0BAB, (230010)_4 = 708 \\
\text{(iv)} & 0p_11 \rightarrow L p_100 & AB\bar{B}B & (0111)_2 = 7 & LBB00, (00033)_4 = 15 \\
\text{(v)} & 1p_00 \rightarrow L 1H0 & BABA & (1010)_2 = 10 & RAAAA0, (21113)_4 = 599 \\
\text{(vi)} & 1p_01 \rightarrow L p_111 & BAB\bar{B} & (1011)_2 = 11 & LBB11, (00022)_4 = 10 \\
\text{(vii)} & 1p_10 \rightarrow R 1p_00 & B\bar{B}\bar{B}A & (1111)_2 = 14 & R1B\bar{B}A, (220010)_4 = 634 \\
\text{(viii)} & 1p_11 \rightarrow L p_110 & B\bar{B}\bar{B}B & (1111)_2 = 15 & LBB10, (00023)_4 = 11 \\
\end{array}
\]

In row (ii) the exponents of \( d \) is calculated to be 14, in order to simulate the move stated in column A. First we have to calculate where to put the information.
This is done coding 0p₀1 in the following way. Replace 0, p₀ and 1 by A, AB and B, respectively. Then we get the word AABB in column B. This word is interpreted as a binary number which is calculated in column C to be 3. This means that the information about the move is to be located between the c number 4 and 5. The L in the word LBB01 means that the head moves to the left in this move. The rest B01 of the word LBB01 codes the word p₁01, where p₁ is replaced by BB and the rest of the word is kept unchanged. Then LBB01 is interpreted as a base 4 number in the following way: L, B, 0 and 1 are interpreted as the digits 0, 0, 3 and 2, respectively. The result is (00032)₄ which is the decimal number 14.

![Universal Turing machine: UTm](image)

**Fig. 1. Universal Turing machine: UTm**

![Turing machine example: Tme](image)

**Fig. 2. Turing machine example: Tme**

In row (iii) the columns A, B and C are made in the same way as in row (ii). In column D the word R0BAB represents the subword 01p₀ of column A in the following way. R means that the move is a right move. 0 is kept unchanged. 1 is