An Introduction to Dynamic Labeled 2-structures

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Abstract. The notion of a dynamic labeled 2-structure is introduced and investigated. It generalizes the notion of a labeled 2-structure (L2s), see [ER1], by making it possible to change the (label) relationships between the nodes. This is achieved by storing in the nodes of a L2s output and input functions which can change the outgoing and incoming labels, respectively. The notion of a clan which is central in the theory of L2s's is transferred to the framework of dL2s's, and the basic properties of clans of dL2s's are investigated.

Introduction

The theory of 2-structures forms a convenient framework for considering various kinds of formal structures encountered in mathematics and computer science (see, e.g., [ER1], [ER2], and [ER4]). A labeled 2-structure, abbreviated L2s, is a finite domain D together with a (labeling) function λ from the set of all 2-edges over D into some alphabet Δ (a 2-edge over D is an ordered pair of different elements of D). Hence a L2s g = (D,Δ,λ) may be seen as representing a set of nodes D together with the relationships between all pairs of different nodes, those relationships are given by λ. Such a L2s is static in the sense that the relationships given by λ are given once and
forever - they cannot change. This may be a disadvantage in modeling systems
that are dynamic, where the relationships between elements of a system may
change during the evolution of a system; graph grammars and computer networks
are examples of systems of such a nature.

In this paper we introduce the notion of a dynamic labeled 2-structure,
abbreviated d\(\xi\)2s. It consists of two components:
(i) a set of nodes \(D\) where in each node \(x\) a set of output functions \(O_x\) and a
set of input functions \(I_x\) is stored; this component is called a mutating
scheme,
and
(ii) a set \(G\) of \(\xi\)2's with \(D\) as the common domain, where \(G\) is closed w.r.t.
transformations of the 2-edges induced by output and input functions in the
nodes of \(D\).

Applying an output function \(\varphi\) in a node \(x\) changes the current label
\(\lambda(x,y)\) of each outgoing 2-edge \((x,y)\) into \(\varphi(\lambda(x,y))\), and similarly, applying
an input function \(\gamma\) in a node \(x\) changes the current label \(\lambda(y,x)\) of each
incoming 2-edge \((y,x)\) into \(\gamma(\lambda(y,x))\).

Clearly, in order to make the above setup workable (e.g., what happens to
the label \(\lambda(x,y)\) if "simultaneously" an output function is applied in \(x\) and an
input function is applied in \(y\)?) one needs to make some basic assumptions
about the underlying mutating scheme. To this aim we will give a set of four
axioms and consider only mutating schemes satisfying this set of axioms - such
mutating schemes are called simply transitive.

It turns out that (simply transitive) d\(\xi\)2s's have an elegant mathematical
structure. First of all we prove that the sets of output (input) functions in
all nodes are equal, and moreover each of them is a group. Then we prove
that also the set of labels with a suitably chosen operation forms a group \(\Delta\),
and applying an output function \(\varphi\) in a node \(x\) amounts to the left
multiplication in \(\Delta\) of labels \(\lambda(x,y)\) by a specific symbol \(a\) associated with \(\varphi\),
while applying an input function \(\gamma\) in a node \(x\) amounts to the right
multiplication in \(\Delta\) of labels \(\lambda(y,x)\) by a specific symbol \(b\) associated with \(\gamma\).
Thus the investigation of d\(\xi\)2s's happens to large extent within the framework
of the theory of groups.

In this paper we demonstrate how to transfer some of the central notions
of the theory of \(\xi\)2s's such as reversibility and clans into the framework of
d\(\xi\)2s's. In particular we show that the notion of reversibility carries
over quite naturally to d\(\xi\)2s's using involutions on the group of labels. Also,
the notion of a clan carries over naturally to the framework of d\(\xi\)2s's. Using
a basic technique of transformations w.r.t. horizons we prove some basic
properties of clans, and in particular we prove that the set of clans of a
d\(\xi\)2s has strong closure properties.