Target Signatures for Maxwell's Equations*

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1 Introduction

Inverse scattering problems can be broadly divided into two distinct classes. The first of these assumes minimal a priori knowledge of the scattering object and attempts to reconstruct the shape of an obstacle or the function values of constitutive parameters from an inexact knowledge of the far field pattern. The mathematical basis for this class of problems is extensively discussed in my recent monograph with Rainer Kress ([6]). The second class of inverse scattering problems is concerned with the determination of 'target signatures,' i.e. eigenvalues that can be determined from the scattering data that are characteristic of the scattering object but not the incident field. In this case, the purpose is not to reconstruct the scattering obstacle or constitutive parameters but rather to either distinguish a specific object from a known set of objects or to detect the existence of an anomaly in a given background configuration. Typical of this second class of inverse scattering problems is the singularity expansion method as discussed in [2] and [8]. The purpose of this short survey talk is to acquaint the reader with some recent (and not so recent) results on target signatures associated with the scattering of electromagnetic waves by a bounded obstacle.

I shall begin my talk by recalling the salient features of the singularity expansion method. Although this method is well known to electrical engineers in connection with radar applications ([2]) and to mathematicians in connection with the mathematical theory of scattering (c.f. the epilogue to [10]), there seems to be very little communication between the two groups (an exception is [8]). The basic idea of the singularity expansion method is to determine the rate of decay of the scattered electric field with respect to time. This rate of decay is determined by the scattering frequencies associated with Maxwell's equations defined in the exterior of a scattering obstacle, i.e. the target signatures are the scattering frequencies. Due to the difficulty in actually measuring the rate of decay of the scattered field, the only scattering frequencies of much practical interest are those nearest the origin.

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The singularity expansion method requires a broad range of frequencies (corresponding, for example, to a broad band pulse as an incident field) for its practical application. By contrast, Colton and Monk ([7]) and Colton and Kirsch ([4],[5]) have recently introduced a new set of target signatures which is applicable for a single fixed frequency. This set is determined by the fact that a convex combination of the electric and magnetic far field patterns for time harmonic fields with arbitrary orthogonal polarizations and arbitrary directions is incomplete for a discrete set of the convexity parameter \( \gamma \). This set of parameters \( \{\gamma_j\} \) can be used as a set of target signatures. Since the accuracy of the determination of the set \( \{\gamma_j\} \) is enhanced by increasing the number of incident fields and the number of points at which the far field pattern is measured, the set of target signatures \( \{\gamma_j\} \) is probably more suitable for an area such as nondestructive testing rather than the usual radar applications. On the other hand, it is undoubtedly premature to make any firm statements on applicability at this stage since so far only a few preliminary experiments concerning the calculation of the set \( \{\gamma_j\} \) from synthetic data have been made ([7]). In addition, the mathematical investigation of the set \( \{\gamma_j\} \) has just begun. A striking result which has been discovered is that for an imperfect conductor the set \( \{\gamma_j\} \) is contained in a region in the complex \( \gamma \)-plane whose geometry depends only on the surface impedance of the scattering obstacle ([5]).

2 The Singularity Expansion Method

Let \( \mathcal{E}, \mathcal{H} \) be the electric and magnetic fields in the exterior of a perfectly conducting scattering obstacle \( D \subset \mathbb{R}^3 \), i.e. after suitable normalization \( \mathcal{E} \) and \( \mathcal{H} \) satisfy Maxwell's equations

\[
\begin{align*}
curl \mathcal{E} + \frac{\partial \mathcal{H}}{\partial t} &= 0 \\
\text{div} \mathcal{E} &= 0
div \mathcal{H} &= 0
\end{align*}
\] (2.1)

in the exterior of \( D \) and

\[
\nu \times \mathcal{E} = 0
\] (2.2)
on the boundary \( \partial D \) of \( D \) where \( \nu \) is the unit outward normal to \( \partial D \). The singularity expansion method is based on the observation that under certain assumptions on \( D \) (e.g. \( D \) is 'nontrapping') \( \mathcal{E} \) has the local asymptotic expansion

\[
\mathcal{E}(x,t) \sim \sum_{j=1}^{\infty} a_j e^{-ik_j t} E_j(x)
\] (2.3)

for \( x \in \mathbb{R}^3 \setminus \bar{D} \) where the \( a_j \) are constants (or possibly polynomials in \( t \) and the \( k_j \) and \( E_j \) are the eigenvalues and eigenfunctions of the non-self adjoint eigenvalue problem

\[
\begin{align*}
curl E_j - ik_j H_j &= 0 \\
\text{curl} H_j + ik_j E_j &= 0
\end{align*}
\] (2.4)