A Directional Path Distance Model for Raster Distance Mapping

Cixiang Zhan, Sudhakar Menon and Peng Gao

Environmental Systems Research Institute
380 New York Street
Redlands, CA 92374 , USA

Abstract. This paper proposes a directional model for weighted path distance mapping on raster data structures. The approach that is used to map the shortest path distance from a set of source cells to any background cell incorporates both non-directional and directional weights on the links between adjacent cells. Non-directional weights allow users to model impedance in terms of the traditional cell based friction costs used in raster distance mapping. Directional weights allow users to model non-symmetric travel costs incurred in traveling through value-gradients in continuous fields such as elevation, temperature or density, or incurred when traveling through a prevailing flow field. The model has been implemented as a cell-based raster distance mapping tool with user selectable directional factor functions, using both Dijkstra's and the Bellman-Ford's shortest distance algorithms. Two experimental results are included.

1 Introduction

Distance is a fundamental variable in spatial operations involving movement or regionalization based on spatial proximity. In a broad sense, distance can be viewed as a relation defined on a set of objects in a space, and can be represented by a relation matrix, in which the diagonal entries are zero, and others, if valid, are positive values [Gatrell, 1983]. In order for its value to be unique at every entry in the matrix distance must be a one-to-one mapping from an *ordered* pair of objects to a non-negative value. Although this notion of distance does not provide the mapping function to calculate distances, it allows great flexibility in defining a variety of distances.

The Euclidean distance between two objects fits this distance concept. The Euclidean distance between two objects requires no path for its definition, and is based solely on the locations of the two objects. The Euclidean distance between two objects is symmetric, i.e. $D(x, y) = D(y, x)$, and satisfies triangularity, i.e. $D(x, z) \leq D(x, y) + D(y,z)$.

The path distance from an object to another requires paths between the two objects. Since paths from an object to another may not be unique and distance along those paths generally are different, path distance does not satisfy the unique distance concept. The shortest path distance from one object to another, however, is unique, and satisfies this distance concept. Smith [1989] shows that the shortest path distance also

satisfies triangularity. We will refer to the shortest path distance as the path distance in cases where there is no confusion.

The path distance between two objects may not just be the physical distance along the path. To each segment of a path, a cost per-unit of physical distance or length, such as gasoline consumption or time, may be attached. The weight attached to a segment of path may be directional or non-directional. By non-directional weights we mean that the cost per-unit length is independent of travel directions along the segment. By directional weights we mean that the cost per-unit length is dependent on travel direction along the segment. Without weights or with non-directional weights, the path distance is symmetric. When directional weights are imposed, the path distance becomes non-symmetric.

The task of distance mapping on a cell-based raster data structure, given a set of *source* objects (represented as sets of cells sharing a unique cell value against a background value) consists of computing, for each cell in the cell layer, the shortest distance from the set of objects to the cell. Note that in the presence of directional weights the value computed for a cell is not necessarily the shortest distance from the cell to the set of objects. Various distance mapping algorithms have been developed for the Euclidean and non-directional path distance mapping problems (Danielsson [1980], Tomlin [1980] [1990], Borgefors [1986], Eastman [1989], Menon et al [1991]). However, the problem of path distance mapping with directional weights on a raster layer has received much less attention.

Directional weights exist widely in actual situations, and in many cases can not be simplified as non-directional weights. The cost to travel up a sloping route is certainly different from that to travel down on the same route, and heat flows from locations of high temperature to location of low temperature. Driving a vehicle against the wind direction consumes more fuel than driving along the wind direction, and particles disperse most easily along the flow direction. In this paper, we describe a general model for weighted path-distance mapping on a cell-based data structure, which considers both directionally uniform weights such as friction cost, and directionally heterogeneous weights that can be modeled as resulting from value gradients in fields such as density, elevation and temperature, and/or prevailing flow directions.

2 Raster Directional Path Distance Model

In order to compute the shortest path distance in the cell-based raster data structure, we need to define objects, valid paths (links) between objects and distance formulae. The objects in a cell layer can be generally defined as all single cells with the location of a cell represented simply by its center. However, a set of cells with the same value, either connected or non-connected can also be viewed as a single object, and the distance to the object from a cell is the shortest distance from the center of the cell to the center of the nearest cell in the object. The valid paths from any cell to other cells must pass through the links from the center of the cell to the centers of its eight