Conformally Stationary Cosmological Models

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Abstract: The role played by conformally stationary space-times in Cosmology is discussed.

Let us consider a general property of the conformally stationary space-times due to Tauber-Weinberg [1] and Ehlers-Geren-Sachs [2]:

There exists an observer \( n \) measuring an isotropic distribution function solution of the Liouville equation if, and only if, the space-time admits a timelike conformal Killing vector \( \xi \). Moreover, \( \xi \) and \( n \) are collinear.

This property is applicable to any relativistic gas. In particular, an isotropic cosmic microwave background radiation can exist only if the space-time is conformally stationary. And this radiation appears isotropic only with respect to the observer associated with the conformal Killing vector.

On the other hand, the redshift \( z \) in a conformally stationary space-time is given by\(^1\)

\[
1 + z = \sqrt{\frac{g_{00}(x_r)}{g_{00}(x_e)}}
\]

where \( x_r \) and \( x_e \) are, respectively, the reception and emission events conected by a null geodesic. Substituting (1) in the following relation\(^2\)

\[
T_r = \frac{T_e}{1 + z}
\]

where \( T \) is the effective radiation temperature, one has

\(^1\) this expression is exactly the same as the redshift formula in a stationary space-time. Here \( g_{00} = g(\xi, \xi) \), \( g \) being the space-time metric.

\(^2\) Note that (2) is valid for a general state of radiation whose distribution function obeys the Liouville theorem.
If one admits the hypothesis that the background radiation was isotropic at decoupling time, the emission temperature is given by

$$T_e = \frac{C}{\sqrt{-g_{00}(x_e)}}$$

where $C$ is a constant. Then (3) and (4) imply

$$T_r = \frac{C}{\sqrt{-g_{00}(x_r)}}$$

that is to say, this radiation is isotropic now. Otherwise, if the background radiation is anisotropic now with respect to the observer associated to the conformal Killing vector, then these anisotropies are primordial.\(^4\)

Therefore, we can have isotropic radiation in an inhomogeneous Friedmann perturbation, assuming that it is conformally stationary. This prompted us to reconsider an important result in Cosmology known as the Sachs-Wolfe effect \([3]\): inhomogeneities in the last scattering surface the Friedmann universe introducing a gravitational potential, which this one changes the energy of photons as a gravitational redshift. In this way one predicts microwave background anisotropies related to the variations of the gravitational potential over the last scattering surface. Recently \([4]\), \([5]\), we have proposed to revise this conclusion mainly because one can see that the model of universe used by Sachs-Wolfe (see below) is just a conformally space-time.

If we restrict ourselves to conformally static space-times, the metric may be written in the form

$$ds^2 = -\alpha^2(x^\mu)(dx^0)^2 + \Omega^2(x^\mu)\gamma_{ij}(x^k)dx^i dx^j$$

where $\frac{\alpha}{\Omega}$ is independent of $x^0$, the coordinate time adapted to the integrable conformal Killing vector, and $(x^\mu) = (x^0, x^i), i = 1, 2, 3$.

Elsewhere \([6]\), we have obtained the expression of the energy-momentum tensor of the metric (6). Generically, it is inhomogeneous and has associated anisotropic pressures. Moreover, the energy density flux (with respect to $n$) is not zero. Then, this flux could be interpreted\(^5\) as due to the relative motion of the matter with respect to the observer measuring isotropic radiation. Thus, an observer comoving with the matter will measure a dipolar anisotropy.

As an example, let us consider a conformally static potential perturbation to the Einstein-de Sitter universe,

$$ds^2 = a^2(\eta)[-(1 - 2\phi)d\eta^2 + (1 + 2\phi)\delta_{ij}dx^i dx^j]$$

\(^3\) This is a consequence of the aforementioned Ehlers-Geren-Sachs results \([2]\)

\(^4\) i.e. these anisotropies were already present at decoupling time.

\(^5\) Under the general algebraic conditions.