Robot Mapping: Foot-Prints vs Tokens

(Extended Abstract)

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Abstract: We are interested in the problem of robot exploration from the approach of competitive analysis, where the cost of an online-strategy is compared with the minimum cost carrying out the same task with perfect information.

Our world model is restricted to graph maps. Within this model, there are several differences dealing with different robot sensors. Most often robots are assumed to have perfect vision. In this paper, however, we consider two different sensors: tokens and foot-prints. For the former, robots cannot recognize nodes or edges of the unknown graph under exploration but can drop some tokens which can be recognized if it returns to nodes where tokens are dropped. In the latter case, the robot has the power of knowing whether a node or an edge has been visited before, though it may not remember exactly when and where it was visited (similar to a traveler lost in the desert who recognizes its foot-print, or a robot smells its own trace.)

With competitive analysis, we want to minimize the ratio of the total number of edges traversed for mapping the graph divided by the optimum number of edge traversals for verifying the map. In particular, we call a strategy competitive if this ratio is constant. As a first step, we have developed a competitive strategy to map an unknown embedded planar graph with pure foot-prints. Then we apply this technique to obtain an algorithm using \( n \) identical tokens to competitively map unknown planar embedded graphs. We also give a lower bound of competitive ratio \( \Omega(n) \) for mapping general embedded graphs with a single token, when robot strategies are slightly restricted. This is tight since there is an algorithm of competitive ratio \( O(n) \) [DJMW].

1 Introduction

We are interested in designing algorithms for a robot to build a map of the environment, based on its own observations in exploring the world. Graph maps are used here to represent the world of robots. Thus, the robot can traverse edges to move from nodes to nodes in the graph. Exploration strategies are evaluated by the ratio of the cost of building the map, where we know nothing

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about the world initially, and the cost of verifying the map, where we have a map of the world and the initial position-orientation of the robot in the map, but still want to verify the correctness of the given information. This approach, called competitive analysis [ST], is used as a measure of information-efficiency of algorithms, for robot navigations and explorations [BCR, BBFY, BRS, DKP, DP, FFKRRV, K1, KP, KRR, PY]).

Different built-in sensors result in different perceptions of the world by the robot. Usually, it is assumed that nodes or edges traversed before can be completely recognized. In contrast, it is assumed in [RS] that nodes are divided into a small number of classes, e.g., white and black colors, and can only be recognized as such. They take the approach of Valiant learning model to design algorithms which correctly infer the environment with high probability. An even more extreme assumption is made in [DJMW] that nodes cannot be recognized at all, as a result of errors in robot's estimation of its location, in sensor reading, in estimation of distance moved between measurements. (See also [KB, LL] for some other qualitative formulations of exploration problems.) To recognize the environment, tokens are used in [DJMW] for the robot to drop at nodes or pick up from nodes while exploring. Thus, by dropping one token at a node, we can recognize the node when we return to find the token there. We may also consider the mapping problem from a desert traveler's viewpoint: We may not recognize a node (or an edge) when we return but we know whether it was previously visited by recognizing our own foot-print.

We want to design an efficient strategy for a robot to obtain the map of an unknown embedded graph \( G = (V, E) \). The embedding establishes a (circular) order of the edges incident to a node. Using the entrance edge, this circular order can be broken into a linear one. Each edge may lead to a new node not yet visited. Nodes can be given names the first time they are reached. However, when reaching a node that is already visited, we have to find out its name which was given the first time the node was reached. We also need to identify which node each edge incident to a given node leads to. We call this the mapping problem. Since the graph is unknown, the cost of mapping is dependent on both the strategy and the unknown graph. The competitive ratio of a strategy is defined as the maximum ratio of the number of traversed edges to establish the map divided by the minimum number of edges traversed for verifying the map. Obviously, we want to design a strategy with the minimum competitive ratio. Notice that, for robots with perfect sensors, mapping is the same as the graph traversal problem. However, when this ultimate power is deprived of, mapping a graph becomes much more different from the traversal problem of the graph. We may not know the graph even after all the edges are traversed!

There are some connections between these different models. For example, for a graph of \( n \) nodes and \( m \) edges, if we have \( n + m \) distinct tokens, we can put one token at one node or edge. This would reduce the token model to the perfect sensor model. Also we can use \( \binom{n}{2} + \binom{m}{2} \) identical tokens to simulate distinct tokens by using \( i \) identical tokens in place of a distinct token with label \( i \). It is well-known that, with perfect sensors, we can explore a general graph