Universal Quasi-Prime Algebraic Domains
(Extended Abstract)

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Summary. This paper demonstrates the existence of a saturated quasi-prime algebraic domain. It also presents a cpo of quasi-prime generated information systems for solving domain equations.

Key Words: Domain theory, semantics of programming languages, universal structures, lattices, universal algebra.

1 INTRODUCTION

Quasi-prime algebraic domains are a class of cpos within the Scott domains. They are introduced by the author in [11] as a new domain-theoretic model for linear logic. Quasi-prime algebraic domains with quasi-linear functions form a monoidal closed category. The unique

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characteristic of the category is that the morphisms are not 'linear', as the term 'quasi-linear' suggests. This is a bit surprising, since all other known domain theoretic linear categories all use linear functions as morphisms [11].

However, how robust and useful the concept of quasi-prime algebraic domains is depends on whether or not they have other nice domain theoretic properties. One of the desirable properties is the existence of a universal (or even saturated) domain [4] in a certain category. The other related property to have is a framework for solving domain equations by fixed point construction, as in [5, 8]. It is the purpose of the paper to establish these results for quasi-prime generated information systems which represent quasi-prime algebraic domains.

One of the most useful results on universal domains is given in the work of Droste and Göbel [1], who introduced the Fraissé-Jónsson theorem in model theory into the area of domain theory. This makes it much easier to show the existence of certain universal domains because it reduces the existence of a saturated structure to the amalgamation property of the finite objects of a certain category.

We apply the result of Droste and Göbel for showing the existence of a saturated (universal, homogeneous) quasi-prime algebraic domain. Our main definition here is the notion of q-embeddings for quasi-prime algebraic domains. The appropriate notion of embeddings for Scott domains (call them s-embeddings) [5, 2] and for dI-domains (call them r-embeddings – 'r' for rigid) [8] are well-known. However, none of these embeddings works for quasi-prime algebraic domains, for the following reasons:

- The s-embeddings are too general: under this embedding the colimit of an ω-chain of finite Scott domains (which are quasi-prime algebraic) need not be quasi-prime algebraic, because any Scott domain can be seen as a colimit of this kind.