Liveness in Bounded Petri Nets which are Covered by T-Invariants

Kurt Lautenbach and Hanno Ridder

University Koblenz-Landau
Institute for Software Technology
Rheinau 1
56075 Koblenz, Germany
E-Mail: {laut, ridder}@informatik.uni-koblenz.de

Abstract: In this paper a criterion is introduced that is sufficient for the liveness in Petri nets which are bounded and covered by non-negative T-invariants.

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1 Introduction

Petri nets are well known models for the representation and analysis of distributed systems (see, e.g. [Rei91, Bau90]). The net itself reflects the topological structure of the system, whereas the variability of the markings represents the dynamic behaviour.

The liveness of a marking, i.e. the fact that every transition can be enabled again and again, is a property that is as important as formally hard to treat.

In contrast to marked graphs, where a marking is live iff all non-negative S-invariants are marked, nets with shared places show a certain "lack of structure" w.r.t. all known methods to investigate liveness. Of course, there is the famous liveness criterion for extended free choice nets according to which a net is live iff all its deadlocks contain a marked trap. But here the lack of structure is compensated by the fact that the structure of extended free choice nets is "relatively simple".

Continuing this way of loosely speaking about net structures, the method we introduce in this paper consists of adding structure to the nets without changing their dynamic properties.

In detail, by adding of so-called regulation circuits to a net we get a new net consisting of only one (elementary) T-invariant. This new net has two desirable properties. Firstly, by increasing the marking of the regulation circuits sufficiently, the new net becomes an arbitrarily exact approximation of the original net. Secondly, the structure of one-T-invariant nets is rich enough for liveness investigations. So, in order to test a net for liveness we examine wether it can be approximated by live one-T-invariant nets. The idea of transforming nets into one-T-invariant nets is not new (cf. [Lau77, KL82, CCS91]). What is new
is to exploit consequently the possibility of approximating the original nets by the one-T-invariant nets.

The class of bounded nets which are covered by T-invariants guarantees a considerable modelling power. All systems with a system wide ability to reproduce situations and without an unlimited increase of the number of objects can be modelled by means of such nets. From the view point of net theory it is important that this class contains a large class of non-free-choice nets.

The paper is organized as follows. Section 2 contains basic definitions and notations, in particular S- and T-invariants, deadlocks and traps. In section 3 the concept of a controlled deadlock is introduced. Controlled deadlocks cannot get unmarked. The mechanism that prevents a controlled deadlock from getting unmarked is quite different from a marked trap inside the deadlock. Next, we deal with liveness in one-T-invariant nets. These nets have the property of being live iff they are weakly live, and sufficient for the weak liveness is that no deadlock can be emptied. Finally, we treat the liveness of bounded nets covered by T-invariants by approximating these nets by one-T-invariant nets.

2 Basic Definitions and Notations

This section contains the basic definitions and notations which will be needed in the rest of the paper.

2.1 Place/Transition Nets

Definition 1.

1. A net is a triple $\mathcal{N} = (S, T, F)$ with
   (a) $S$ and $T$ are finite, nonempty and disjoint sets.
   (b) $F \subseteq (S \times T) \cup (T \times S)$.
   The elements of $S$ are called places and the elements of $T$ transitions.

2. The preset of a node $x \in S \cup T$ is defined as $\mathcal{R} = \{ y \in S \cup T | (y, x) \in F \}$.
   The postset of $x \in S \cup T$ is $\mathcal{P} = \{ y \in S \cup T | (x, y) \in F \}$. The preset (postset) of a set is the union of the presets (postsets) of the elements.

Definition 2. Let $\mathcal{N} = (S, T, F)$ be a net.

1. A marking of a net $\mathcal{N} = (S, T, F)$ is a mapping $M : S \rightarrow \mathbb{N}$.
2. The pair $(\mathcal{N}, M)$ is called a net system or marked net; $M$ is called the initial marking.
3. A transition $t \in T$ is called enabled under $M$, in symbols $M[t]$, iff
   $\forall s \in \mathcal{R} : M(s) \geq 0$.
4. If $M[t]$, the transition $t$ may occur, resulting in a new marking $M'$, in symbols $M[t]M'$, with
   $M'(s) = \begin{cases} 
   M(s) - 1 & \text{if } s \in \mathcal{R} \setminus \mathcal{P} \\
   M(s) + 1 & \text{if } s \in \mathcal{P} \setminus \mathcal{R} \\
   M(s) & \text{otherwise}
   \end{cases}$
   for all $s \in S$. 