Time-Optimal Nearest-Neighbor Computations on Enhanced Meshes

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Abstract. A simple polygon P is said to be unimodal if for every vertex of P, the Euclidian distance function to the other vertices of P is unimodal. The study of unimodal polygons has emerged as a fruitful area of computational and discrete geometry. It is well-known that nearest and furthest neighbor computations are a recurring theme in pattern recognition, VLSI design, computer graphics, and image processing, among others. Our contribution is to propose time-optimal algorithms for constructing the Euclidian Minimum Spanning Tree, the Relative Neighborhood Graph, as well as the Symmetric Further Neighbor Graph of an n-vertex unimodal polygon on meshes with multiple broadcasting. We begin by establishing a $\Omega(\log n)$ time lower bound for solving arbitrary instances of size n of these problems. This lower bound holds for both the CREW-PRAM and for the mesh with multiple broadcasting. We obtain our time lower bound results for the CREW-PRAM by using a novel technique involving geometric constructions. These constructions allow us to reduce the well-known OR problem to each of the geometric problems of interest. We then port these time lower bounds to the mesh with multiple broadcasting using simulation results.

Next, we show that the time lower bound is tight by exhibiting algorithms for these tasks running in $O(\log n)$ time on a mesh with multiple broadcasting of size $n \times n$.

Keywords: unimodal polygons, mesh with multiple broadcasting, pattern recognition, nearest-neighbor norms, computational geometry, computational morphology, parallel algorithms, EMST, RNG, SFNG

1 Introduction

A mesh-connected multiprocessor array of size $M \times N$ consists of $MN$ identical SIMD processors positioned on a rectangular array, with every processor having its own local memory of size $O(\log MN)$. The processor $P(i, j)$ is located in row i and column j ($1 \leq i \leq M; 1 \leq j \leq N$) with $P(1, 1)$ in the north-west corner of the mesh. Every processor is connected to its four neighbors, provided they exist, and is assumed to know its own coordinates within the mesh.

Unfortunately, the mesh is slow when it comes to handling data transfer operations over long distances. To overcome this problem, mesh-connected computers have recently been augmented with various types of bus systems [3]. For example, Aggarwal [1] and Bokhari [10] have considered mesh-connected multiprocessors enhanced by the addition of $k$ global buses. Yet another such system that is commercially available [25] involves enhancing the mesh architecture by the addition of row and column buses as shown in Figure 1. In [15] an abstraction of such a system is referred
being of theoretical interest as well as commercially available, the mesh with multiple broadcasting has attracted a great deal of well-deserved attention. In the last years, efficient algorithms to solve a number of computational problems on meshes with multiple broadcasting have been proposed in the literature. These include image processing [16, 25], computational geometry [6, 7, 15, 20, 23, 24], semigroup computations [8, 15], selection [5, 15], among others.

A PRAM [14] consists of synchronous processors, all having unit-time access to a shared memory. At each step, every processor performs the same instruction, with a number of processors masked out. In the CREW-PRAM, a memory location can be simultaneously accessed in reading but not in writing.

A mesh with multiple broadcasting can be perceived as a restricted version of the CREW-PRAM: the buses are nothing more than oblivious concurrent read, exclusive write registers with the access restricted to certain sets of processors. Indeed, a square mesh with multiple broadcasting using $p$ processors can be viewed as a CREW-PRAM with $p$ processors where groups of $\sqrt{p}$ of these have concurrent read access to a register whose value is available for one time unit, after which it is lost. Given that the mesh with multiple broadcasting is, in this sense, weaker than the CREW-PRAM, it is very often quite a challenge to design algorithms in this model that match the performance of their CREW-PRAM counterparts. Typically, for the same running time, the mesh with multiple broadcasting uses more processors. This phenomenon will appear in our algorithms.

In pattern recognition and classification, the shape of an object is routinely represented by a polygon obtained from an image processing device [4, 13, 30]. One of the fundamental features that contributes to a morphological description useful