New Advances in Laplacian Growth Models

F. Guinea and O. Pla¹, E. Louis² and V. Hakim³

¹ Instituto de Ciencias de Materiales, CSIC, Cantoblanco, E-28049 Madrid, Spain
² Departamento de Física Aplicada, Universidad de Alicante, Apdo. 99, E-09080 Alicante, Spain
³ Laboratoire de Physique Statistique, Ecole Normale Superiéure, 24 Rue Lhomond, 75231 Paris CEDEX 05, France

1 Introduction

In 1983, the model called Diffusion Limited Aggregation was first introduced in the scientific literature[1, 2], in order to study the formation of very tenuous structures, such as soot and dust. Since its conception, a large number of variations have been discussed, and applied to phenomena as varied as lightning or electrolytic deposition. The underlying feature which can be used to unify such diverse growth phenomena is the existence of a field which satisfies Laplace's equation, and which determines the evolution of the system.

The vast potentiality of Laplacian growth models was developed in a series of successive steps. Among others, it is worth mentioning:

- The generalization to models unrelated to aggregation proper, but controlled by a Laplacian field: the Dielectric Breakdown Model[3].
- The understanding of its relation to the growth process par excellence, solidification of an undercooled liquid and dendritic formation[4].
- The connexion of DLA to other 'classic' problem, viscous fingering[5].
- The generalization of DBM to elastic problems and crack formation[6].
- The application to the varied morphology found in electrolytic deposition[7].

While the advances in relating Laplacian growth to different physical processes has been rather spectacular, theoretical understanding progresses at a much slower pace. Some basic features were quickly identified, as crucial in the formation of the fractal patterns found in numerical simulations: the long range effects and screening mediated by laplacian field, and the tip splitting instability of the branches of the patterns. The latter instability dissapears in he presence of anisotropy and in the absence of noise. These are always the conditions when the growing pattern is a crystal. Stable branch tips lead to dendritic growth. It is also assumed that the global shape is determined by some kind of competition between the various branches which from it.

The DLA model itself has been the object of extensive numerical simulations, and the resulting patterns have been characterized in great detail[8]. Numerical
work in many of the related models is significantly more difficult, as it requires the calculation of the laplacian field throughout the entire space[9].

The model has also proven to be quite hard to tackle by the standard methods of statistical physics. Different attempts to define dome kind of mean field approach have been tried, with limited success[10]. A variety of renormalization group approaches have been used. The most promising schemes focuses on the details of the growth of a particular branch, which then are 'blown up' to the entire aggregate[11]. While this technique describes well the global features, the way in which the scale invariance of the model is generated is left unresolved. The extensive mathematical knowledge of the 2D Laplace equation has also been applied. It was found that, by means of conformal mapping methods, the model allows for an elegant formulation. A number of general properties, conserved quantities and even exact solutions have been found in this manner[12]. It is yet unclear, however, whether the average global features can be obtained by this technique.

Thus, laplacian growth remains a challenging theoretical problem, and its elucidation would surely clarify many issues in the physics of systems evolving far out of equilibrium. To show its broad range of applicability, we show in figures (1) and (2) a pattern obtained by electrolytic deposition (courtesy of J. M. Pasror, M. A. Rubio and E. Crespo (UNED, Madrid), and a similar pattern found in the fossil record (from A. G. Checa and J. M. Garcia-Ruiz, Ammonoids paleobiology, N. Chapman ed. Plenum, 1994).

It is impossible in a short article to review in full the many new developments which are being proposed. In the following sections, we will address two topics which are representative of the many approaches to the problem available in the literature. We discuss:

- The variation in shapes of the Dielectric Breakdown Model as function of the growth law. The goal is to find 'simple' limits which allow us to use analytical perturbative schemes. While this objective has not been achieved, a variety of

Fig. 1. Patterns obtained by electrolytic deposition.