On Transforming Intuitionistic Matrix Proofs into Standard-Sequent Proofs

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Abstract. We present a procedure transforming intuitionistic matrix proofs into proofs within the intuitionistic standard sequent calculus. The transformation is based on L. Wallen's proof justifying his matrix characterization for the validity of intuitionistic formulae. Since this proof makes use of Fitting's non-standard sequent calculus our procedure consists of two steps. First a non-standard sequent proof will be extracted from a given matrix proof. Secondly we transform each non-standard proof into a standard proof in a structure preserving way. To simplify the latter step we introduce an extended standard calculus which is shown to be sound and complete.

1 Introduction

According to the proofs-as-programs paradigm theorems proven in a constructive manner can be interpreted as specifications of programs which are contained in the proof. Therefore proof tools for constructive logics are very important for the development of verifiably correct software. Because of the expressiveness of the underlying calculus these tools are essentially interactive proof editors supported by a tactic mechanism for programming proofs on the meta-level. On the other hand theorem provers like Setheo [9], Otter [16], or KoMeT [3] show that reasoning about classical predicate logic can be automated sufficiently well. It would therefore be desirable to have a procedure which automatically generates the purely logical parts of a proof during a session with a proof editor for a rich constructive theory. This would liberate its user from having to deal with rather tedious but boring parts of the proof. The proof created by such a procedure should be expressed within the calculus underlying the proof development tool to allow the extraction of programs.

Proof editors like the NuPRL system [4] are based on a sequent calculus supporting the construction of proofs which are comprehensible for mathematicians and programmers. It includes a calculus for predicate logic similar to Gentzen's calculus for intuitionistic logic [8]. This calculus, which contains at most one formula in the succedent of a sequent, will be considered a standard sequent calculus $\mathcal{LJ}$.

In [15] L. Wallen successfully created a matrix characterization $\mathcal{M}\mathcal{J}$ for the validity of intuitionistic formulae. His theoretical framework is based on Fit-
ting's [5] non-standard sequent calculus $LJ_{NS}$ which allows the occurrence of more than one succedent formula. Because of this characterization it is possible to construct the purely logical parts of a NuPRL-proof in two steps. First a matrix proof in $MJ$ has to be found by some effective proof procedure. Wallen suggested extending Bibel's connection method [1, 2] for this purpose. Secondly the matrix proof has to be transformed back into a valid standard sequent proof.

In this paper we shall focus on the second step, i.e. on a procedure transforming a proof which was derived efficiently in $MJ$ into a proof within the standard sequent calculus $LJ_{S}$. Because of Wallen's investigations we can be sure that such a $LJ_{S}$-proof must exist but there is not yet an efficient method to construct it from a a given $MJ$-proof. In order to do this we again proceed in two steps. First we represent the $MJ$-proof in the non-standard calculus $LJ_{NS}$. Secondly we convert the resulting non-standard sequent proof into a standard proof. We will show, however, that because of the strong differences between the rules of the two calculi it is not possible to transform every $LJ_{NS}$-proof into a corresponding $LJ_{S}$-proof without changing the proof structure. To solve this problem we have developed an extended standard calculus $LJ_{S}^{E}$ which is able to represent each $LJ_{NS}$-proof in a structure preserving way. We have proven $LJ_{S}^{E}$ to be sound and complete and implemented its rules as tactics of the NuPRL system. Therefore we can transform intuitionistic matrix proofs into extended NuPRL proofs without any additional search.

In the following section we shall briefly review the sequent calculi $LJ_{S}$ and $LJ_{NS}$ and summarize the notation which is necessary to understand Wallen's matrix characterization $MJ$. Section 3 will discuss the procedure transforming matrix proofs into non-standard sequent proofs. In section 4 we shall present an $LJ_{NS}$-proof which cannot be converted into an equivalent $LJ_{S}$-proof in a structure preserving way and introduce the extended standard calculus $LJ_{S}^{E}$. Section 5 will present the transformation from $LJ_{NS}$ into $LJ_{S}$. We conclude with a few remarks on implementation issues and efficient search procedures for $MJ$-proofs.

2 Preliminaries

Our transformation procedure relates intuitionistic proofs in three entirely different calculi: a matrix characterization $MJ$ [15], a non-standard sequent calculus $LJ_{NS}$ [5], and the standard sequent calculus $LJ_{S}$ [8]. In this section we shall briefly review these calculi.

Wallen's basic idea for developing the matrix characterization $MJ$ which we shall describe below was to use Schütte's embedding of intuitionistic logic $J$ into the modal logic $S4$ [12] together with the Kripke semantic similarity between $S4$ and $J$ [13]. Therefore his investigations were based on an intuitionistic sequent calculus which has a structure similar to the one for $S4$. Contrary to Gentzen's (standard) sequent calculus $LJ_{S}$ this calculus allows more than one formula in the succedent of a sequent. We therefore call it a non-standard calculus and denote it by $LJ_{NS}$. A proof for its correctness and completeness can be found