Modeling Uncertain Relational Knowledge:
the AV-Quantified Production Rules Approach

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Abstract
Relational knowledge is a very common form of knowledge, which allows direct inferences to be drawn from a premise to a conclusion. This paper focuses on the problem of representing and using relational knowledge affected by uncertainty. We first discuss the intuitive meaning of the uncertainty that may affect relational knowledge and we distinguish between A-uncertainty, concerning the applicability of a relation, and V-uncertainty, concerning the validity of a relation. Then we show how the difference between A-uncertainty and V-uncertainty has received so far only limited attention in various literature proposals. Finally, we introduce an original approach to deal with uncertain relational knowledge based on AV-quantified production rules, and we discuss its main features.

1 Introduction
One of the main features of intelligent behaviour is the ability to derive properties (called conclusions) about certain aspects of the world, inferring them from the available information relevant to different, but related, aspects (called premises). The simplest form of knowledge that can be used to this purpose consists in direct associations (relations) between the truth values attributed to a pair of propositions, one representing the premise and the other the conclusion of the inference process. For the sake of simplicity, we assume that propositions can assume just two truth values, i.e. true and false.

In many cases, the task of drawing an inference from a premise to a conclusion has to be accomplished in presence of uncertainty affecting both the truth value of the premise and the relation between the premise and the conclusion.

This paper focuses on the problem of representing and using uncertainty about the relation between the premise and the conclusion.

2 The Meaning of Uncertainty Affecting a Relation between Propositions
Given a proposition (expressing a fact about the world) and its possible truth values (supposed to be mutually exclusive), to be in a state of uncertainty about that proposition simply means to ignore which truth value has to be assigned to the proposition (given the hypothesis that the proposition has a truth value in the real world). The meaning of the term uncertainty is not so univocal and clear when dealing with uncertainty affecting (binary) relations between propositions. In fact, as we show below, uncertainty about a relation can carry two different meanings, depending on whether it affects the applicability or the validity of the relation.

Uncertainty about the applicability of a relation
This case is well exemplified by default rules [Reiter 80]. Here uncertainty arises from the fact that there are exceptions to these rules and that it is practically impossible to
enumerate and explicitly represent all the exceptions. For instance, given the default rule "birds fly", exceptions are represented by all the birds that do not fly, such as penguins, ostriches, emus ... (and broiled birds, according to [Pearl 91]). Default rules may be viewed as rules with a weak, incomplete premise, that fails to capture all the conditions under which the rule is applicable. The meaning of the default rule "birds fly" is therefore not exactly

\[
\text{if is-a(X, bird) then fly(X)}
\]

but, more appropriately:

\[
\text{if is-a(X, bird) and "conditions"(X) then fly(X).}
\]

Often, we are unable (or unwilling) to specify all the "conditions" that make the rule applicable, since we have only incomplete knowledge about the relation existing between birds and the property of flying. However, we are intuitively certain about the validity of the rule, since in general, apart some exceptions, birds fly. "Birds fly" is in a sense an incomplete (or imprecise) rule, but it is definitely certain in nature. As it is evident, for any bird you meet - say Tweety - if you were able to correctly decide whether it meets "conditions" or not, you would able to deduce as well, without doubt, whether Tweety can fly or not. However, if you can not decide whether "conditions" are met or not - what is actually the case being "conditions" unknown - on the basis of the common experience that most birds fly, you may in any case assume that Tweety flies, until specific evidence against this assumption becomes available. Uncertainty only affects the applicability of the rule, not its validity. This kind of uncertainty does not arise from a lack of evidence supporting the validity of the rule, but from the inherent incompleteness of its premise. We are simply unable to articulate all the conditions (that indeed exist) that make the rule applicable, either because they are too many and too intricate or because they are (partially) unknown. We call this type of uncertainty that affects the applicability of a relation, but not its validity, A-uncertainty.

**Uncertainty about the validity of a relation**

In order to examine this case, let us consider the following two examples:

- "inveterate smokers catch lung cancer": it is definitely proved that smoke is a cause of lung cancer, but of course not all inveterate smokers necessarily catch lung cancer - statistically, an inveterate smoker has only 85% probabilities of contracting lung cancer;
- "inveterate smokers catch atherosclerosis": the causes of atherosclerosis are not exactly known, the majority of scholars (60%) indicates smoke as a probable cause of atherosclerosis, whereas others (40%) exclude this hypothesis.

The first rule represents a chunk of knowledge which is not really affected by uncertainty: there is no doubt that a (causal) relation exists between smoke and lung cancer. However, if your friend Tom is an inveterate smoker and you apply this knowledge to Tom, you can not be sure that Tom is among the 85% of smokers who will suffer from cancer. To put it another way, it is possible to consider the presence of inhibitors which prevent smoke from causing cancer and have a 15% statistical