Dualities between Nets and Automata Induced by Schizophrenic Objects *

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Abstract. The so-called synthesis problem for nets, which consists in deciding whether a given graph is isomorphic to the case graph of some net, and then constructing the net, has been solved in the literature for various types of nets, ranging from elementary nets to Petri nets. The common principle for the synthesis is the idea of regions in graphs, representing possible extensions of places in nets. When the synthesis problem has a solution, the set of regions viewed as properties of states provides a set-theoretic representation of the transition system. We show that such correspondences between nets and transition systems can be described as dualities induced by schizophrenic objects, leading further the analogy with classical representation theorems and giving us a means to describe uniformly previously known translations between nets and automata.

1 Introduction

The so-called synthesis problem for nets, which consists in deciding whether a given graph is isomorphic to the case graph of some net, and then constructing the net, has been solved in the literature for various types of nets, ranging from elementary nets to Petri nets. The common principle for the synthesis is the idea of regions in graphs, representing possible extensions of places in nets. When the synthesis problem has a solution, the set of regions viewed as properties of states provides a set-theoretic representation of the transition system where transition systems and nets can be viewed respectively as the extensional versus the intensional description of discrete event systems. Namely, a state of a transition system can be represented by the set of properties it satisfies, and an event which is given in extension by a set of transitions between states may also be described intensionally by the set of properties it modifies. Now the fact that an event is enabled in a state can be deduced from the representations of the event and the state.

In order to illustrate these points let us recall the synthesis problem for elementary net systems. In the formalism of elementary net systems, properties are figured out by places, and an event $a$ is encoded by a pair of disjoint sets of places $\langle \bullet a, a\bullet \rangle$. The properties in $\bullet a$ (the preconditions of $a$) are necessary

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conditions for event $a$ to proceed and they no longer hold after this event has occurred. Symmetrically, properties in $a^*$ (the postconditions of $a$) never hold in states where the event $a$ is enabled, and always hold after the execution of $a$.

The transition system associated with an elementary net, the so-called *state graph* of the net, thus consists of those transitions $M \xrightarrow{a} M'$ where $M$ and $M'$ are sets of places (markings) such that $M \setminus M' = \cdot a$ and $M' \setminus M = a^*$. This can be rephrased as

$$M \xrightarrow{a} M' \iff \cdot a \subseteq M \land a^* \cap M = \emptyset \land M' = (M \setminus \cdot a) \cup a^*$$

i.e. an event is enabled at every marking that contains all its preconditions and none of its postconditions, and the new marking is obtained by withdrawing preconditions and adding postconditions. An elementary net system is an elementary net together with a distinguished initial marking $M_0$. The synthesis problem consists in deciding whether a given finite automaton $A = (Q, A, T, q_0)$ is isomorphic to the state graph of some elementary net system, i.e. to the restriction of the state graph of the underlying net accessible from its initial marking.

An automaton which is isomorphic to the state graph of some elementary net system is called an elementary transition system. Let $A = (Q, A, T, q_0)$ be an elementary transition system and $NS = (P, A, \cdot(), ()^*, M_0)$ be an elementary net system whose state graph is isomorphic to $A$, where $P$ is the set of places, the mappings $\cdot(), ()^* : A \to 2^P$ indicate respectively the preconditions and postconditions of each event $a \in A$, and $M_0 \subseteq P$ is the initial marking. Since each state of the automaton $A$ may be identified with a corresponding marking of the elementary net system $NS$, one can define a binary relation $\models \subseteq Q \times P$ between states of $A$ and places of $NS$, where $q \models x$ (read "the state $q$ satisfies the property/place $x$") when the marking associated with the state $q$ contains the place $x$. The elementary net $N = (P, A, \cdot(), ()^*)$ provides a set-theoretic representation of the transition system $TS = (Q, A, T)$, in the sense that we have a pair of mappings $\cdot_Q : Q \to 2^P$ and $\cdot_A : A \to 2^P \times 2P$ defined as $\cdot_Q q = \{ x \in P \mid q \models x \}$ (hence $\cdot_Q q$ is the marking associated with $q$) and $\cdot_A a = \langle \cdot a, a^* \rangle$ such that (i) the representation is faithful, i.e. both mappings are injective and (ii) the transition relation in $TS$ is characterized by

$$q \xrightarrow{a} q' \in T \iff q' \cdot_Q \setminus q \cdot_Q = \cdot a \land q' \cdot_A \setminus q \cdot_A = a^*$$

In order to construct such a representation for a given transition system $TS$ one should guess the adequate set of places (tokens of the representation). For that purpose we proceed the other way round: we suppose that such a net $N = (P, A, \cdot(), ()^*)$ exists and we represent each place $x \in P$ by the set $\cdot_Q x = \{ q \in Q \mid q \models x \}$