The Weak Late \( \pi \)-calculus Semantics as Observation Equivalence*

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Abstract. We show that the Weak Late \( \pi \)-calculus semantics can be characterized as ordinary Observation congruence over a specialized transition system where both the instantiation of input placeholders and the name substitutions, due e.g. to communication, are explicitly handled via suitable constructors. The approach presented here allows to axiomatize the Weak Late \( \pi \)-calculus semantics by simply adding Milner's \( \pi \)-laws to the proof system for the Strong equivalence. Resorting to Observation equivalence provides a framework which is general enough to allow to recover, in straightforward ways, other bisimulation semantics (e.g. Early, both Strong and Weak, and Dynamic and Branching, both Early and Late).

0 Introduction

This paper aims at contributing to the understanding of the Weak \( \pi \)-calculus bisimulation semantics [Mil90]. The \( \pi \)-calculus [MPW92] is an instance of value-passing process algebra where values are names, and then communication causes substitution of names for names.

In [FMQ94] we proposed an alternative formulation of the Strong \( \pi \)-calculus semantics. There, contrary to the original definition of the calculus, name substitutions were explicitly handled via the introduction of a suitable state operator. That approach allowed the characterization of the \( \pi \)-calculus transitional semantics as ordinary (i.e. CCS-like [Mil80]) labelled transition system, where the usual notion of Strong bisimulation applies.

We claim that reasoning about the \( \pi \)-calculus semantics in terms of ordinary bisimulation may be of both theoretical and pragmatic interest: General theorems of the format-based theory may be applied (e.g. those proved in [DS85, ABV94]), and algorithms for CCS automated verification tools may be re-used (e.g. [CPS93]). This paper provides a further step towards accomplishing the task of interpreting the \( \pi \)-calculus semantics as ordinary bisimulation. We generalize the techniques adopted in [FMQ94] in order to fairly deal with the

* Work partially supported by ESPRIT BRA Project 6454 CONFER, and by Progetto Coordinato CNR 'Strumenti per la Specifica e la Verifica di Proprietà Critiche di Sistemi Concorrenti e Distribuiti'.
\(\pi\)-calculus Weak bisimulation equivalences which, ignoring the silent actions representing internal communications, have surely more practical interest than their Strong duals. We discuss in full detail the Weak Late bisimulation semantics, and show how to retrieve other semantics by minor changes.

According to the Late \(\pi\)-calculus semantics, the fact that names can be transmitted in interactions shows up in a 'functional' operational intuition about input actions. Precisely, the transition \(P_1 \xrightarrow{\pi(y)} P_1'\) means that the process \(P_1\) evolves into \(P_1'\) by inputting any name, formally called \(y\), over the channel named \(z\). The name \(y\) is only a placeholder. It stands for any name \(w\) whose input would force \(P_1\) to proceed as \(P_1'(\{w/y\})\), namely as \(P_1'\) where all the free occurrences of \(y\) are replaced by \(w\), with suitable \(\alpha\)-conversions to avoid name clashes. For instance, when \(P_1\) runs in parallel with the process \(P_2 = \overline{w}.P_2'\), which can output \(w\) on \(x\) and then behave like \(P_2'\), the communication is expressed by the transition \(P_1 \parallel P_2 \xrightarrow{\tau} P_1'(\{w/y\}) \parallel P_2'\).

The functional intuition about input actions has an impact on behavioural equivalences, and particularly on the Weak bisimulation. Letting \(\leftrightarrow\) be the reflexive and transitive closure of \(\rightarrow\), and \(\overset{\alpha}{\rightarrow}\) be \(\rightarrow\) \(\leftarrow\rightarrow\), and \(\overset{\hat{\alpha}}{\rightarrow}\) be \(\rightarrow\) if \(\alpha = \tau\), and \(\overset{\alpha}{\rightarrow}\) otherwise, the ordinary Weak bisimulation clause is as follows [Mil89]:

\[(W)\] if \(P \overset{\alpha}{\rightarrow} P'\) then for some \(Q', Q' \overset{\overset{\alpha}{\rightarrow}}{\rightarrow}\), and \(P'\) is Weak bisimilar to \(Q'\).

The Late interpretation of input actions breaks the above double arrow in two, and the bisimulation clause becomes [Mil90]:

\[(WL)\] if \(P \overset{\pi(y)}{\rightarrow} P'\) then for some \(Q', Q' \overset{\overset{\pi(y)}{\overset{\alpha}{\rightarrow}}}{\overset{\hat{\alpha}}{\rightarrow}}\), and for all \(w\) there exists \(Q''\) such that \(Q' \overset{\overset{\pi(y)}{\overset{\alpha}{\rightarrow}}}{\overset{\hat{\alpha}}{\rightarrow}} Q''\) and \(P' \overset{\{w/y\}}{\rightarrow} Q''\) is Weak Late bisimilar to \(Q''\).

The universal quantification on \(w\), and the claim that the instantiation of \(y\) takes place just after the input move, make \((WL)\) dramatically different than \((W)\), and suggests that the actual instantiation of the input placeholder can be assigned an operational meaning.

In this paper we show that the Weak Late \(\pi\)-calculus semantics can be characterized as ordinary Weak equivalence (also called Observation equivalence) over a specialized transition system where the placeholder instantiation becomes a distinguished move. Intuitively, the input move \(\bullet \overset{\pi(y)}{\rightarrow} \bullet\) is rendered as something like the following: