Abstract. The two-dimensional Danish Eulerian Model has been developed during the 80's. More than 70 chemical reactions (some of them photochemical) are involved in the model. The space domain of the model covers the whole of Europe. It has been discretized by using a \((32 \times 32)\) equidistant spatial grid. The concentrations and the species calculated by the model were compared both with measurements taken over land and with measurements taken over sea. The model has also been run by using finer grids; as, for example, a \((96 \times 96)\) grid. The experiments indicated that in general the model calculates rather reliable results. However, it is also clear that the results might be improved if a three-dimensional version of the model is developed. Three-dimensional air pollution models are very time-consuming. Therefore the development of a reliable three-dimensional version of an air pollution model is a very challenging task. The efforts to solve some of the numerical problems arising during the development of a large three-dimensional air pollution model (with non-linear chemical reactions) will be discussed in this paper.

1 Need for large air pollution models

Air pollutants emitted by different sources can be transported, by the wind, on long distances. Several physical processes (diffusion, deposition and chemical transformations) take place during the transport. Regions that are very far from the large emission sources may also be polluted. It is well-known that the atmosphere must be kept clean (or, at least, should not be polluted too much). It is also well-known that if the concentrations of some species exceed certain acceptable (or critical) levels, then they may become dangerous for plants, animals and humans.

Mathematical models are needed in the efforts to predict the optimal way of keeping the air pollution under acceptable levels. It should be emphasized here
that the mathematical models are the only tool by the use of which one can predict the results of many different actions and, moreover, one can attempt to choose the best solution (or, at least, a solution which is close to the best one) in the efforts to reduce the air pollution in an optimal way. The mathematical models are necessarily large (the transport of air pollutants is carried out over long distances and, thus, the space domains are very large).

2 The Danish Eulerian Model

The work on the development of the Danish Eulerian Model has been initiated in 1980. First a simple transport scheme was developed (one pollutant only and without chemical reactions). The next step was the development of a simple model with two pollutants (and linearized chemical reactions). An experimental model containing ten pollutants and non-linear chemical reactions (including here photochemical reactions) was the third step. The first operational version of the Danish Eulerian Model was based on a chemical scheme with 35 pollutants. Some experiments with chemical schemes containing 56 and 168 pollutants are carried out at present. Different versions of the Danish Eulerian Model are discussed in [3] and [6].

The reliability of the operational model with 35 pollutants has been tested by comparing model results both with measurements taken over land ([4], [5]) and with measurements taken over sea ([2]). Test-problems, where the analytical solution is known, have been used to check the accuracy of the numerical algorithms ([3]).

3 Mathematical description of the Danish Eulerian Model

The Danish Eulerian model is described by a system of PDE's (the number of equations in this system being equal to the number of pollutants involved in the model):

\[
\frac{\partial c_s}{\partial t} = -\frac{\partial (uc_s)}{\partial x} - \frac{\partial (vc_s)}{\partial y} - \frac{\partial (wc_s)}{\partial z} + \frac{\partial}{\partial x} \left( K_x \frac{\partial c_s}{\partial x} \right) + \frac{\partial}{\partial y} \left( K_y \frac{\partial c_s}{\partial y} \right) + \frac{\partial}{\partial z} \left( K_z \frac{\partial c_s}{\partial z} \right) \\
- (\kappa_{1s} + \kappa_{2s}) c_s + E_s + Q_s(c_1, c_2, \ldots, c_q), \quad s = 1, 2, \ldots, q. \tag{1}
\]

The different quantities that are involved in the mathematical model have the following meaning:
- the concentrations are denoted by \( c_s \);
- \( u, v \) and \( w \) are wind velocities;
- \( K_x, K_y \) and \( K_z \) are diffusion coefficients;
- the emission sources in the space domain are described by the functions \( E_s \);
- \( \kappa_{1s} \) and \( \kappa_{2s} \) are deposition coefficients;