Relative Normalization in Deterministic Residual Structures

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Abstract. This paper generalizes the Huet and Lévy theory of normalization by neededness to an abstract setting. We define Stable Deterministic Residual Structures (SDRS) and Deterministic Family Structures (DFS) by axiomatizing some properties of the residual relation and the family relation on redexes in an Abstract Rewriting System. We present two proofs of the Relative Normalization Theorem, one for SDRSs for regular stable sets, and another for DFSs for all stable sets of desirable 'normal forms'. We further prove the Relative Optimality Theorem for DFSs. We extend this result to deterministic Computation Structures which are deterministic Event Structures with an extra relation expressing self-essentiality.

1 Introduction

A normalizable term, in a rewriting system, may have an infinite reduction, so it is important to have a normalizing strategy which enables one to construct reductions to normal form. It is well known that the leftmost-outermost strategy is normalizing in the $\lambda$-calculus [CuFe58].

For Orthogonal Term Rewriting Systems (OTRSs), a general normalizing strategy, called the needed strategy, was found by Huet and Lévy [HuLé91]. The strategy always contracts a needed redex - one whose residual has to be contracted in any reduction to normal form. Huet and Lévy showed that any term not in normal form has a needed redex, and that repeated contraction of needed redexes leads to its normal form whenever there is one.

This work has been extended in several directions. Barendregt et al. [BKKS87], Maranget [Mar92], and Nöcker [Nök94] study neededness w.r.t. head-normal forms, weak head-normal forms, and constructor head-normal forms, respectively. Sekar and Ramakrishnan [SeRa90] study normalization via necessary set of redexes. Kennaway et al. [KKSV96] study a needed strategy for infinitary OTRSs. A different approach to normalization is developed in Kennaway [Ken89] and Antoy and Middeldorp [AnMi94]. Antoy et al. [AEH94] design a needed narrowing strategy.

In [GlKh94], the present authors address the question of normalization relative to a desired set of final terms, considering the properties that a set of terms must possess in order for the neededness theory of Huet and Lévy still to make sense. This work is

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done in the context of orthogonal Expression Reduction Systems (OERS) [Kha92], a form of higher-order rewriting which subsumes Term Rewriting and the $\lambda$-calculus. Natural conditions are imposed on $S$, called stability, that are necessary and sufficient for the following Relative Normalization (RN) theorem to hold: each $S$-normalizable term not in $S$ (not in $S$-normal form) has at least one $S$-needed redex, and repeated contraction of such redexes will lead to an $S$-normal form whenever there is one. It is shown also that if a stable $S$ is regular, i.e., if $S$-unneeded redexes cannot duplicate $S$-needed ones, then the $S$-needed strategy is hypernormalizing as well.

In this paper, we further generalize the theory by abstracting from the structure of terms. We study relative normalization in Deterministic Residual Structures (DRSs). A DRS is an Abstract Reduction Systems (ARSs) which has a residual relation between redexes in the source and target terms of each transition. Redexes of $t$ may be erased by reduction of $u$, new redexes may be introduced in $s$, while other redexes of $s$ are considered residuals of redexes in $t$, as specified by the residual relation. Further, the residual relation is generalized to all reductions, and permutation-equivalence on reductions, referred to below as Lévy-equivalence, and the embedding relation, which induces a partial ordering on the reduction space, is introduced, as is done for the $\lambda$-calculus in [Lév78, Lév80]. Sufficient conditions needed to define the above concepts in an abstract setting were stated in [Sta89, GLM92].

In [Sta89], Stark defines Concurrent Transition Systems (CTSs) and uses them to develop a model of concurrent computation, studying ways of building machine networks with concurrent machines as basic objects. On the other hand, Gonthier et al. [GLM92] were interested in studying more syntactic properties, such as standardization, of orthogonal rewrite systems in an abstract setting. The way standardization is understood in that paper requires a nesting relation on redexes in a term, and some axioms giving its important properties. Standard reductions then become some kind of outside-in reductions. However, the [GLM92] axioms are rather restrictive, since even orthogonal DAGs [Mar91, Mar92, KKS93] do not satisfy them as pointed out by R. Kennaway.

Our DRSs are more refined than CTSs, since in the latter the residual relation is non-duplicating. We do not impose a nesting relation on redexes, but are still able to prove the RN theorem for all regular stable sets $S$. (We actually prove the Relative Hypernormalization theorem.) We use a form of Berry's stability axiom [Ber79] and show that without this axiom the theorem fails. The proof method employed is similar to that in [Kha88, Kha93], and is based on the fact that $S$-needed steps in a reduction can be pushed before $S$-unneeded steps without affecting the number of $S$-needed steps. The important difference is that [Kha88, Kha93] uses the syntactic notion of descendants of subterms — a refinement of the residual notion for redexes — which is much harder to axiomatize.

Since for irregular stable $S$, $S$-unneeded redexes can duplicate $S$-needed ones, the above proof method does not apply for all DRS; for the same reason, the $S$-needed strategy is no longer hypernormalizing. We define a Deterministic Family Structure (DFS) as a DRS with a very liberal notion of family relation [Lév78, Lév80] and a contribution relation on families, expressing the notion of (at least one member of) a family to be needed to create another family. For DFSs, the proof of the RN theorem for all stable $S$ from [GLKh94] works perfectly.

An advantage of the first RN theorem is that checking for Berry's stability