Some Combinatorial Models for Course Scheduling

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Abstract

Timetabling problems have often been formulated as coloring problems in graphs. We give formulations in terms of graph coloring (or hypergraph coloring) for a collection of simple class-teacher timetabling problems and review complexity issues for these formulations. This tutorial presentation concludes with some hints on some general procedures which handles many specific requirements.

1. Introduction

Course scheduling problems provide an excellent example of real problem for which O.R. methods were considered as the most natural techniques to use. Occurring by their nature in an academic world, these problems have expectedly been tackled by a huge variety of tools depending on the particular interests and fields of competence of the potential solvers. Our purpose is not to give a general survey of the various models which have been proposed but rather to focus our attention on combinatorial models and at the same time to show how extensions of classical models have been worked out to formalize and solve some type of timetabling problems. We will also present whenever it will be possible the limits between easy and difficult problems.

Timetabling problems are numerous: they differ from each other not only by the types of constraints which are to be taken into account, but also by the density (or the scarcity) of the constraints; two problems of the same "size", with the same types of constraints may be very different from each other if one has many tight constraints and the other has just a few. The solution methods may be quite different, so the problems should be considered as different.

Here we shall start from the simplest problems and introduce requirements consecutively while discussing the variations of the corresponding combinatorial models. Since they are more structured, we shall put the emphasis on class-teacher timetabling.

In fact, we shall essentially deal with timetabling problems as chromatic scheduling problems, i.e., we will concentrate on graph coloring models. After having presented node coloring in graphs or hypergraphs and edge coloring in graphs, we will very briefly mention some general approaches which may handle many types of requirements.
To keep the paper within a reasonable length, we shall not give standard definitions of graphs, they can be found in [1]. Similarly for concepts related to complexity, the reader is referred to [5].

2. The basic model: class-teacher timetabling problem

We shall start with the simplest situation which occurs in timetabling, the so called class-teacher timetabling problem. A class $c_i$ will consist of a set of students who follow exactly the same programme. $\mathcal{C} = \{c_1, \ldots, c_m\}$ will be the set of classes while $\mathcal{T} = \{t_1, \ldots, t_n\}$ is the set of teachers. A requirement matrix $R = (r_{ij})$ gives the number $r_{ij}$ of (one-hour) lectures involving $c_i$ and $t_j$ for all $i,j$.

The problem consists in constructing (when possible) a timetable in $p$ periods. Each class (resp. teacher) is involved in at most one lecture at a time.

It is well known that a solution exists if and only if

$$\sum_i r_{ij} \leq p \text{ for all } j \text{ and } \sum_j r_{ij} \leq p \text{ for all } i$$

(see [14]).

The problem may be viewed as constructing a generalized Latin square (GLS); a $(p \times p)$ Latin square (LS) is an array in which each cell $(i,j)$ contains one of the $p$ symbols $\alpha, \beta, \ldots, \varphi$ and no symbol appears more than once in any line (row or column). If each symbol is associated with a period, we may consider that the LS corresponds to a requirement matrix $R$ where $r_{ij} = 1$ for each class $c_i$ and each teacher $t_j$; cell $(i,j)$ contains $\delta$ if $c_i$ and $t_j$ meet at period $\delta$. Given an arbitrary $R$, a timetable corresponds to a GLS: cell $(i,j)$ contains $r_{ij}$ symbols in $\{\alpha, \beta, \ldots, \varphi\}$; they are the periods where the meetings of $c_i$ and $t_j$ are scheduled; each symbol occurs at most once in each row and in each column. (Notice that a GLS need in fact not be square!) Figure 1 shows a requirement matrix $R$, an associated GLS and an extended GLS which is square and contains the initial one. It is always possible to transform a rectangle GLS into a larger square GLS which contains the first one, as can be seen easily.

Another formulation would be to associate to $R$ a hypergraph $H(R) = (X, \mathcal{S})$ constructed as follows: each unit in $R$ is a node $x$ in the node set $X$; for each line (row or column) of $R$ we introduce an edge containing all nodes associated to units in this line. An example is given in Figure 2.