Weak Bisimulation and Model Checking for Basic Parallel Processes

Richard Mayr

Institut für Informatik, Technische Universität München, Arcisstr. 21, D-80290 München, Germany; e-mail: mayrri@informatik.tu-muenchen.de

Abstract. Basic Parallel Processes (BPP) are a natural subclass of CCS infinite-state processes. They are also equivalent to a special class of Petri nets. We show that unlike for general Petri nets, it is decidable if a BPP and a finite-state system are weakly bisimilar. To the best of our knowledge, this is the first decidability result for weak bisimulation and a non-trivial class of infinite-state systems. We also show that the model checking problem for BPPs and the branching time logic UB is PSPACE-complete. This settles a conjecture of [4].

Keywords: Basic Parallel Processes, bisimulation, model checking.

1 Introduction

Bisimulation equivalence [10], has become one of the most successful equivalence notions in concurrency theory, both with respect to theoretical research and to applications. In recent years, the decidability of bisimulation equivalence on several classes of infinite-state systems has been intensely studied, and a number of positive results have been obtained. Strong bisimilarity between

- two context-free processes,
- two Basic Parallel Processes (BPPs),
- a Petri net and a finite-state system

has been shown to be decidable [3, 2, 8].

A natural next step in this line of research is to extend at least some of these encouraging results to weak bisimilarity, an equivalence that allows to ignore silent internal actions. However, this task has proved to be difficult, and to the best of our knowledge only undecidability results have been obtained so far (i.e. weak bisimilarity of a Petri net and a finite-state system is undecidable [7]). In this paper we report on a first positive result: weak bisimilarity between a BPP and a finite-state system is decidable.

In the second part of the paper we shift our attention to the model checking problem for BPPs, called just the model checking problem in the sequel. This is the problem of deciding if a given BPP satisfies a property coded as a formula in a certain temporal logic. It has been shown in [4] that the model checking problem is undecidable for formulae built out of boolean operators, EX (for some
successor) and EG (for some path always in the future), but decidable when the temporal operators are EX and EF (for some path eventually in the future). Therefore, the logic with the latter set of operators, called UB in [4], seems to be the largest branching time logic with a decidable model checking problem. It was shown in [4] that the model checking problem for UB is PSPACE-hard even for finite-state BPPs (notice that the size of the problem is the size of the formula plus the size of the BPP, and not the size of its associated transition system), but the exact complexity of the problem remained open. We prove that this problem is PSPACE-complete by showing that it only requires polynomial space, even for infinite-state BPPs.

2 Preliminaries

Definition 1. A binary relation $R$ over the states of a labelled transition system (for short: LTS) is a strong bisimulation (often simply called bisimulation) iff

$$\forall (s_1, s_2) \in R \forall a \in \text{Act.} (s_1 \xrightarrow{a} s'_1 \Rightarrow \exists s_2 \xrightarrow{a} s'_2. s'_1 Rs'_2) \land (s_2 \xrightarrow{a} s'_2 \Rightarrow \exists s'_1 \xrightarrow{a} s'_1. s'_1 Rs'_2)$$

Two states $s_1$ and $s_2$ are strongly bisimilar iff there is a strong bisimulation $R$ such that $s_1 Rs_2$. This definition can be extended to states in different transition systems by putting them 'side by side' and considering them as a single transition system. It is easy to see that there always exists a largest strong bisimulation which is an equivalence relation called strong bisimulation equivalence. It is denoted by $\sim$.

Strong bisimulation equivalence is sometimes too strict. Processes can contain silent internal actions (labelled by $\tau$) which should not be externally visible. Therefore another equivalence called weak bisimulation equivalence is defined that treats these $\tau$-actions accordingly.

Definition 2. Let $\mathcal{R} := (\tau)^* \xrightarrow{a} (\tau)^*$ for $a \in \text{Act}$ and $\mathcal{R} := \begin{cases} \mathcal{R} \quad &\text{if } a \neq \tau \\ \mathcal{R} \quad &\text{if } a = \tau \end{cases}$

A binary relation $R$ over the states of an LTS is a weak bisimulation if

$$\forall (s_1, s_2) \in R \forall a \in \text{Act.} (s_1 \xrightarrow{a} s'_1 \Rightarrow \exists s_2 \xrightarrow{3\mathcal{R}} s'_2. s'_1 Rs'_2) \land (s_2 \xrightarrow{a} s'_2 \Rightarrow \exists s'_1 \xrightarrow{3\mathcal{R}} s'_1. s'_1 Rs'_2)$$

Two states $s_1$ and $s_2$ are weakly bisimilar iff there is a weak bisimulation $R$ such that $s_1 Rs_2$. Again this can be extended to states in different transition systems. There always exists a largest weak bisimulation which is an equivalence relation called weak bisimulation equivalence. It is denoted by $\approx$. It is clear that $\sim \subseteq \approx$ for every LTS.

A process that is weakly bisimilar to a finite-state LTS is called weakly finite.