On Cooperating Distributed Uniformly Limited 0L Systems

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Abstract. Cooperating distributed grammar systems [1] constitute a formal model of the blackboard model for problem solving. In this paper, we replace the grammars in such systems by uniformly k-limited 0L systems [9]. In this way we can define quite a lot of different language families. We compare these families with each other in respect to inclusion. The connections with other language families and closure properties are also investigated. For instance, it is shown that the new introduced families are incomparable with the families of T0L or uniformly k-limited T0L languages.

1. Introduction

Motivated by the blackboard model of artificial intelligence, Csuhaj-Varjú and Dassow [1] introduced the concept of cooperating distributed grammar systems. In their model, the distributed knowledge sources are represented by grammars, the actual state of the problem corresponds to a sentential form, and the application of some production of a grammar corresponds to an action at the blackboard. The distributed grammars have to cooperate to obtain a solution. The actions at the blackboard are controlled according to different modes. These modes determine how long a certain grammar is allowed to manipulate the sentential forms before giving back control to the system. Cooperating distributed grammar systems and some variants have been also considered in [2], [4] and [5]. A comprehensive text-book presentation is given in [3].

In [9], we have introduced the notion of uniformly k-limited T0L systems (see also [6], [12]). These systems represent a limitation of the parallel rewriting of T0L systems. In short, a uniformly k-limited T0L system (abbreviated as uklT0L system) \( G = (\Sigma, H, \omega, k) \) is given by the limitation \( k \in \mathbb{N} \) (where \( \mathbb{N} \) is the set of natural numbers) and a T0L system \( (\Sigma, H, \omega) \) with alphabet \( \Sigma \), finite set of tables \( H \) (where a table is a finite substitution on \( \Sigma \)), and axiom \( \omega \in \Sigma^* \). A derivation step of a uklT0L system differs from that of a T0L system in such a way that instead of the fully parallel rewriting of L systems, now at each step of the rewriting process, exactly \( \min\{k, |w|\} \) symbols in the word \( w \) considered have to be rewritten (where \( |w| \) is the length of \( w \)). A derivation step from \( w_1 \) to \( w_2 \) according to \( G \) is denoted by \( w_1 \Rightarrow_G w_2 \). If no misunderstanding is possible we write \( \Rightarrow \) instead of \( \Rightarrow_G \). Let \( \Rightarrow^* \) be the reflexive transitive closure of the relation \( \Rightarrow \). Then \( L(G) = \{ w \in \Sigma^* \mid \omega \Rightarrow^* w \} \) is the uklT0L language generated by \( G \). If there is only one table, we talk of a ukl0L system and write \( G = (\Sigma, h, \omega, k) \) where \( h \) is a finite substitution on \( \Sigma \). By \( L(uklT0L) \) and \( L(ukl0L) \), we denote the corresponding families of all uklT0L or ukl0L languages, respectively.
If the derivation mechanism is changed in such a way that at each step of the rewriting process, for every \( a \in \Sigma \) exactly \( \min\{k, \#_aw\} \) occurrences of the symbol \( a \) in the word \( w \) considered have to be rewritten (where \( \#_aw \) is the number of occurrences of the symbol \( a \) in \( w \)), then we get the definition of \( k\text{LTOL} \) and \( k\text{TL} \) systems as introduced in [8].

Every derivation step of a \( k\text{LTOL} \) system (or \( k\text{TL} \) system) has to be carried out with the same limitation \( k \). If we want to change the limitation during the derivation process then we can reach this aim by replacing the grammars of cooperating distributed grammar systems by \( k\text{TL0} \) systems (or \( k\text{LO} \) systems, respectively) with different limitations \( k \).

In the case of \( k\text{L0} \) systems, such cooperating distributed limited 0L systems (CD0L systems) have been already investigated in [10]. Furthermore, in [11] there have been investigated cooperating distributed uniformly-limited 0L systems. The exact definition of such a system (CDu0L system) is given in Section 2. Quite a lot of different language families are defined. In Section 3 we compare CDu0L language families with each other in respect to inclusion. Relative to this aspect we get nice characterizations of the so-called CD(\( k_1, \ldots, k_r \))u0L language families where only some special cases remain open. In Section 4 we compare the CDu0L language families with other families. Especially, all propagating CDu0L languages are context-sensitive. Finally, in Section 5 we shall see that all CDu0L families are anti-AFL’s.

In the sequel, we denote by \( N \) the set of all natural numbers (where \( 0 \notin N \)). Then \( N_0 = N \cup \{0\} \).

2. CDu0L Systems, Definitions and Simple Results

A cooperating distributed uniformly limited 0L system (CDu0L system for short) is a construct

\[ G = (\Sigma, (h_1, k_1), \ldots, (h_r, k_r), \omega) \]

for \( r \in N \) (the number of components of the system), alphabet \( \Sigma \), a word \( \omega \in \Sigma^* \) (the axiom), finite substitutions \( h_\rho \) (the tables of the system) and natural numbers \( k_\rho \in N \) (the limitations) where \( \rho = 1, \ldots, r \). Obviously, \( G_\rho = (\Sigma, h_\rho, \omega, k_\rho) \), \( \rho = 1, \ldots, r \), can be considered as a u0L system. Especially, a system \( G \) as above is also called a CD(\( k_1, \ldots, k_r \))u0L system. If \( r = 1 \), we also write CD\( k_1 \)u0L system. \( G \) is called deterministic if all \( h_\rho, \rho = 1, \ldots, r \), are homomorphisms. \( G \) is called propagating if the empty word \( \epsilon \notin h_\rho(a) \) for all \( \rho \in \{1, \ldots, r\} \) and \( a \in \Sigma \). If \( w \in h_\rho(a) \) for some \( a \in \Sigma \), \( w \in \Sigma^* \), then \( a \rightarrow w \) is called a production of \( h_\rho \). We also talk of the production \( w \in h_\rho(a) \).

Let \( v, w \in \Sigma^* \), \( \rho = 1, \ldots, r \), and \( s \in N \). We write

\[ v \Longrightarrow^*_\rho w \]

if there are words \( w_1, \ldots, w_s = w \in \Sigma^* \) such that there exists a derivation

\[ v \Longrightarrow G_\rho w_1 \Longrightarrow G_\rho w_2 \Longrightarrow G_\rho w_{s-1} \Longrightarrow G_\rho w_s = w \]

according to the uniformly \( k_\rho \)-limited 0L system \( G_\rho \). \( s \) is called the length of the derivation \( v \Longrightarrow^*_\rho w \). We write

\[ u \Longrightarrow^{\leq s}_\rho w \quad (u \Longrightarrow^{\geq s}_\rho w, \text{ respectively}) \]