Four-Valued Diagnoses for Stratified Knowledge-Bases

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Abstract. We present a four-valued approach for recovering consistent data from inconsistent set of assertions. For a common family of knowledge-bases we also provide an efficient algorithm for doing so automatically. This method is particularly useful for making model-based diagnoses.

1 Introduction

It is well-known that the classical calculus allows only trivial reasoning in the presence of inconsistency. This property is particularly problematic when the system under consideration is aimed to deal with conflicts. This is the case, for instance, with diagnostic systems that are supposed to explain the discrepancy between the actual behavior of some device and the way it is meant to behave. A common approach of handling inconsistent information is to consider some consistent subsets that still contain meaningful data. The usual method of doing so is to consider the maximal consistent subsets of the “polluted” data. The main drawback of this method is that none of these subsets necessarily correspond to the intended semantics of the original information. Even in the simplest inconsistent knowledge-base $KB = \{p, \neg p\}$ every maximal consistent subset of $KB$ classically contradicts an explicit data of $KB$. In the case of diagnostic systems this means that a diagnosis based on a maximal consistent subset might not truthfully determine why a given system is not functioning as it was intended. One might, of course, use the intersection of all the maximal consistent subsets. This, however, might be very expensive.

We propose here a different approach to “salvage” consistent data without contradicting any assertion of the original information. Our approach is based on the idea of reducing the number of models by using a second order relation (see details below). For a common family of knowledge-bases we also provide an efficient algorithm for recovering this data. We then illustrate the ideas in a diagnostic system for checking faulty circuits. The underlying formalism is based on Belnap’s four-valued logic \cite{Be77a, Be77b}, and it is nonmonotonic and paraconsistent \cite{dC74} in nature.
2 Preliminaries

We present a formalism that is based on Belnap's well-known four-valued logic. For a detailed discussion on this logic see, e.g., [Be77a, Be77b]. We denote by \( t \) and \( f \) the classical values. \( \perp \) and \( T \) denote, respectively, lack of knowledge and "over"-knowledge (conflict). It is usual to consider these four values according to two partial orders: One, \( \leq_t \), might intuitively be understood as reflecting differences in the "measure of truth" that every value represents. According to this order, \( f \) is the minimal element, \( t \) is the maximal one, and \( \perp, T \) are two intermediate values that are incomparable. \( \{t, f, \perp, T\} \) is a distributive lattice with an order reversing involution \( \neg \), for which \( \neg T = T \) and \( \neg \perp = \perp \). We shall denote the meet and the join of this lattice by \( \land \) and \( \lor \), respectively. The other partial order, \( \leq_k \), is understood (again, intuitively) as reflecting differences in the amount of knowledge or information that each truth value exhibits. Again, \( \{t, f, \perp, T\} \) is a lattice where \( \perp \) is its minimal element, \( T \) - the maximal element, and \( t, f \) are incomparable.

A double-Hasse diagram with the four elements and the two lattices is given in Figure 1 below.

\[ \text{Fig. 1. The four-valued structure} \]

The language we treat here is the propositional language based on \( \{\neg, \lor, \land, \perp, T\} \).\(^1\) Given a set \( S \) of propositional formulae, we shall denote by \( A(S) \) the set of the atomic formulae that occur in \( S \), and by \( L(S) \) the set of the literals that occur in \( S \). The semantic notions are natural generalizations to the four-valued case of similar classical notions: A valuation \( \nu \) is a function that assigns a

\(^1\) \( t \) and \( f \) are definable in this language: \( f = T \land \perp \) and \( t = T \lor \perp \). \( \land \) is of course also definable, using de-Morgan law.