Abstract. For all complexity measures in Kolmogorov complexity the effect discovered by P. Martin-Löf holds. For every infinite binary sequence there is a wide gap between the supremum and the infimum of the complexity of initial fragments of the sequence. It is assumed that this inevitable gap is characteristic of Kolmogorov complexity, and it is caused by the highly abstract nature of the unrestricted Kolmogorov complexity.

We consider the complexity of inductive inference for recursively enumerable classes of total recursive functions. This object is considered as a rather simple object where no effects from highly abstract theories are likely to be met. Here, similar gaps were discovered. Moreover, the existence of these gaps is proved by an explicit use of the theorem by P. Martin-Löf.

In our paper, we study a modification of inductive inference complexity. The complexity is usually understood as the maximum of mindchanges over the functions defined by the first n indices of the numbering. Instead we consider the mindchange complexity as the maximum over the first n functions in the numbering (disregarding the repeated functions). Linear upper and lower bounds for the mindchange complexity are proved. However, the gap between bounds for all n and bounds for infinitely many n remains.
1 Introduction

1.1 Kolmogorov complexity

Consider $K(n)$, $C(n)$ and other functions which measure various versions of Kolmogorov complexity for words, natural numbers and other objects. These functions are not computable. Moreover, even if we are not so much interested in the Kolmogorov complexity of individual objects but we wish only to understand the order of magnitude of the growth of these functions, we discover with some surprise that even functions like

$$K^{max}(n) = \max_{0 \leq i \leq n} K(i)$$

are highly chaotic. The first result of this kind was the theorem by P. Martin Löf [15]. Let $h(n)$ be an arbitrary total recursive function such that the series

$$\sum 2^{-h(n)}$$

diverges. Then for every 0-1 valued function $f$ it is true that for infinitely many values of $n$

$$K_B(f^{[n]}) \leq n - h(n)$$

This theorem showed that there does not exist a maximally complicated binary sequence every initial fragment of which would have the complexity

$$K_B(f^{[n]}) = n$$

If we consider a complicated binary sequence, at the best we have

$$K_B(f^{[n]}) = n$$

for infinitely many values of $n$ but for infinitely many other values of $n$ we have

$$K_B(f^{[n]}) \leq n - h(n)$$

This implies that for all the functions expressing Kolmogorov complexity we are to consider seperately the complexity for nearly all $n$ (or more precisely, for all $n$ but a finite number of them) and the complexity for infinitely many $n$.

Indeed, J. Bärzdins [2] proved many results on Kolmogorov complexity of initial fragments of binary sequences describing recursively enumerable sets, and in all these results the complexity of the recursively enumerable set for nearly all $n$ was essentially higher than the complexity for infinitely many $n$. Various modifications of the problem were considered and various versions of the Kolmogorov complexity were used.

We consider the mindchange complexity in Inductive Inference. This notion seems to be much simpler than the unrestricted Kolmogorov complexity. Hence, one can expect that the gaps between the upper and lower bounds for nearly all $n$ and for infinitely many $n$ may not exist. However they exist! (cf. [4])