9 Turbulence induced particle fragmentation and coalescence

Particles in continuum react differently on the fluctuation of the mean continuum velocity. While bubbles follow quickly the continuum, heavy droplet may considerably delay following the continuum fluctuation. This makes the main difference in the criteria for fragmentation of bubbles and particles. In this Chapter we will give a brief characterization of the homogeneous isotropic turbulence, and then we will analyze the reaction capability of a particle to follow the changes in velocity of the surrounding continuum. We will derive expressions for the maximum relative velocity, which creates distortion and possible fragmentation. Finally we will look for quantitative information describing, as in the previous section, (a) the final diameter after the fragmentation, $D_{\infty}$, and (b) the time interval in which the fragmentation occurs, $\Delta \tau_{br}$. The different components of the turbulent energy dissipation are analyzed. For channel flows expression are given for approximate estimation of the dissipation rate of the turbulent kinetic energy as a function of the frictional pressure drop. The dissipation rate of the turbulent kinetic energy due to relative phase motion is also analyzed. The probability of the bubble and droplet coalescence is then estimated.

9.1 Homogeneous turbulence characteristics

Large scale motion: Observing turbulent motion of continuum one realizes that random eddies overlay the mean flow. The eddies have different size $\ell$ at this length scale. The largest scale of the eddies is limited by the geometrical boundary of the systems. In a pipe flow the natural limit is the pipe diameter

$$\ell \leq D_h. \quad (9.1)$$

Large scale eddies contain the main part of the turbulent kinetic energy. Viscous forces have no effect on the large scale motion. There is no energy dissipation. The large scale motion is characterized by the velocity difference over the scale of the eddies $\ell$, $V'_\ell$. Usually the turbulence Reynolds number

$$Re'_i = \frac{V'_\ell \ell}{\nu} \quad (9.2)$$
is used for characterizing the turbulence. 

**Small scale motion:** Small scale eddies with a size

\[ \ell_c < \ell \]  \hspace{1cm} (9.3)

contains only a small part of the kinetic energy of turbulent motion. There are characterized by

\[ Re'_e = \frac{V'_e \ell_c}{\nu} . \]  \hspace{1cm} (9.4)

**Viscous limit:** The size of an eddy characterized by

\[ Re'_e = \frac{V'_e \ell_0}{\nu} \approx 1 \]  \hspace{1cm} (9.5)

or

\[ \ell_0 \approx \frac{V}{V'_e} \]  \hspace{1cm} (9.6)

is called *inner scale* or *micro scale* of turbulence. Eddies with

\[ \ell < \ell_0 \]  \hspace{1cm} (9.7)

dissipate mechanical energy by viscous friction in internal energy (heat). Fluctuations with such sizes are gradually damped. The very nature of the turbulent motion is the continuous transfer of mechanical energy from larger to smaller eddies.

**Dimensional analysis for small scale motion:** We already stated that the turbulent motion of scale larger than the viscous limit does not depend on viscosity and is characterized by \( V'_e, \, \ell_c, \, \rho \). Starting with this observation, *Kolmogoroff* (1941, 1949) found that the only combination of these flow parameters having dimension of energy dissipated per unit time and per unit volume is \( \rho V'_e^3 / \ell_c \), or per unit time and unit mass

\[ \varepsilon \approx \frac{V'_e^3}{\ell_c} . \]  \hspace{1cm} (9.8)

The mechanical energy of turbulent dissipation per unit time and unit mass of the continuum \( \varepsilon \) is called *turbulence dissipation rate* in the following. Thus the velocity change over the distance \( \ell_c \)

\[ V'_e = (\varepsilon \ell_c)^{1/3} \]  \hspace{1cm} (9.9)

increases with increasing energy dissipation. The characteristic time period of the fluctuation with given size \( \ell_c \) is then