Appendix: Performance of the 2SHI Estimator
Under the Generalised Pitman Nearness Criterion

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1 Introduction

In 1985, Van Hoa proposed a family of 2SHI (two stage hierarchical information) estimators for the coefficient vector of the linear regression model. These 2SHI estimators were demonstrated to dominate in average mean squared errors (MSE) the OLS and the Stein estimators. A number of applications of the 2SHI estimators in empirical economic studies based on static and dynamic regression models where the 2SHI dominance was calculated have also been reported (see Van Hoa 1992a, 1992b, 1993).

In 1990 and 1993, Van Hoa and Chaturvedi extended the 2SHI further and considered a more general family of 2SHI estimators. They obtained the conditions for the dominance of the 2SHI estimator over the OLS and Stein rule estimators under a quadratic loss function. In those studies, the criterion of relative MSE or risks in the sense of Wald was adopted.

The MSE criterion is only one of many criteria that can be used in the studies of this kind. The Pitman nearness criterion is another concept that has been developed and used by several researchers for a comparison of alternative estimators, see Keating and Mason (1985), Rao et al. (1986), Khatree (1987) and Peddada (1987), to cite a few. A special feature of this criterion is that it does not require the existence of the moments of the estimator and is less sensitive to the tail behaviour of the sampling distributions of the estimator. Rao et al. (1986) and Keating and Mason (1988) considered a Generalized Pitman Nearness (GPN) criterion and analysed the performance of the Stein rule estimator in comparison to the Maximum Likelihood Estimator (MLE) for the mean of the multivariate normal distribution using extensive numerical studies. Sen et al. (1989) derived the dominance condition for the Stein rule estimator over the MLE under a GPN criterion (see also Keating and Czitrom (1988) and Mason et al. 1990).

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The main objective of the present paper is to establish the dominance of the 2SHI estimator over the OLS estimator and the Stein rule estimator under a GPN criterion. Since the technique adopted in Sen et al. (1989) is quite involved and leads to fairly complicated expressions, we have adopted a simple methodology based on the small disturbances approximations.

2 The Model and the Estimators

Consider the linear regression model

\[ y = X\beta + \sigma u, \quad (1) \]

where \( y \) is a Tx1 vector of observations on the dependent variable, \( X \) is a Txk matrix of observations on k independent variables with full column rank, \( \beta \) is a kx1 vector of unknown regression coefficients and \( u \) is a Tx1 random vector following a multivariate normal distribution \( N(0, I_T) \) independent of \( X \) and \( \sigma^2 (>0) \) is the disturbance variance.

In 1990 and 1993, Van Hoa and Chaturvedi generalized the work by Van Hoa (1985) and proposed the following family of explicit 2SHI estimators \( \hat{\beta}_h \) for the coefficient vector \( \beta \):

\[ \hat{\beta}_h = \left[ 1 - cw \frac{(1-R^2)}{R^2} - c(1-w)\frac{(1-R^2)}{R^2} \left[ 1 + c^*(1-R^2)/R^2 \right] \right] b, \quad (2) \]

where \( b = (X'X)^{-1}X'y \) is the OLS estimator of \( \beta \), \( R^2 = (b'X'y/y'y) \) is the coefficient of determination corresponding to a no intercept model and \( w (0<w<1) \), \( c(>0) \) and \( c^*(>0) \) are the characterizing scalars.

We can equivalently write the estimators \( \hat{\beta}_h \) as:

\[ \hat{\beta}_h = \left[ 1 - cw \frac{(y-Xb)'(y-Xb)}{b'X'Xb} - c(1-w)\frac{(y-Xb)'(y-Xb)}{(b'X'Xb+c^*(y-Xb)')(y-Xb)} \right] b, \quad (3) \]

It can be verified that, when \( c^*=0 \) or \( w=1 \), the 2SHI estimator \( \hat{\beta}_h \) reduces to the following Stein rule estimator \( \hat{\beta}_s \).

\[ \hat{\beta}_s = \left[ 1 - \frac{(y-Xb)'(y-Xb)}{b'X'Xb} \right] b. \quad (4) \]

3 Comparison of the Estimators

For the comparison of the estimators, let us consider the quadratic loss function