We review recent work based on extending the known relations between Markov processes and Markov semigroups to the case where time is replaced by hypersurfaces of codimension 1 and the state space is a group. Relations with gauge fields, Higgs fields and relativistic fields are also discussed.
1. Introduction

The study of commutative and non commutative stochastic differential equations for processes with values in groups is well developed, and has found applications in particular in the investigation of the gauge groups of mappings of \( \mathbb{R} \) or \( S^1 \) into Lie groups, see e.g. [1], and references therein. Problems of the theory of representations of groups of mappings and non commutative distribution theory, see e.g. [2], as well as problems of the theory of gauge fields suggest the creation of a theory of commutative and non commutative stochastic partial differential equations. E.g. a pure Yang-Mills Euclidean measure gives a white noise type distribution to a curvature 2-form and the corresponding connection 1-form has to be obtained then by "non commutative stochastic integration". In a series of papers [3] we have already given parts of such a theory, see also [4], [15] for other approaches and problems. In this lecture we give a short survey of aspects of our approach, stressing in particular the opportunity to study so called stochastic group-valued measures as basic quantities related to generalized Markov semigroups, in a similar way as Markov processes are related to Markov semigroups. Markov cosurfaces arise as objects which correspond to Markov processes in the case of one-dimensional time, and we stress their relations with stochastic group-valued measures. A Lévy-Khinchine type formula is derived for the latter and recent results by Kaufmann [5] on continuity properties (non commutative higher dimensional time analogs of Prokhorov-Kolmogorov criteria) of Markov cosurfaces are mentioned. Remarks on lattice Gibbsian models, gauge and Higgs fields, relativistic quantum fields and gauge-group representations are also given.

2. Generalized Markov semigroups on groups, and stochastic group-valued measures

Let \((\mathcal{M}, \mathcal{B})\) be a measurable space and let \(G\) be a locally compact group. A stochastic \(G\)-valued (multiplicative) measure \(\eta\) on \((\mathcal{M}, \mathcal{B})\) is a process \(\eta\) indexed by \(\mathcal{B}\) with state space \(G\) so that \(\eta(A)\) for any \(A \in \mathcal{B}\) is a random variable on some probability space \((\Omega, \mathcal{F}, \mathbb{P})\), with \(\eta(\emptyset) = e\), and, for disjoint \(A, B \in \mathcal{B}\), \(\eta(A) \perp \eta(B)\) (where \(\perp\) means independence), as well as \(\eta(A \cup B) = \eta(A) \cdot \eta(B)\), where equality is in law and the product is in \(G\). Moreover we assume that the law of \(\eta(A)\) has an invariant density with respect to the Haar measure on \(G\) i.e. \(\mathbb{P}_{\eta(A)}(h_1 h_2) = \mathbb{P}_{\eta(A)}(h_2 h_1)\), \(h_1, h_2 \in G\), and has some suitable continuity properties (cfr. [3], d,e, [5]).

Remark 1. This is obviously an extension to "times in \(\mathcal{M}\)" of the concept of stochastic measure (random measure) associated with processes with independent stationary increments (i.e. of infinitely divisible type) ([8], [9]).

As in the case of "one-dimensional time" we introduce associated (convolution) semigroups of probability measures, which we call generalized Markov semigroups: such a semigroup is a family of probability measures \(P\) on \(G\) indexed by \(\mathcal{B}\), with the